Mini-Lecture 1.1
Linear Equations

Learning Objectives:

1. Solve a linear equation
2. Solve equations that lead to linear equations
3. Solve problems that can be modeled by linear equations

Examples:

1. $(a)\ 6+3x=9x+6$ $\quad (b)\ 2x-(3x+3)=2x-18$ $\quad (c)\ \dfrac{x+4}{2}+\dfrac{x+1}{3}=10$

2. $(a)\ \dfrac{1}{4}+\dfrac{6}{x}=\dfrac{5}{8}$ $\quad (b)\ x(2x-5)=(2x+2)(x-3)$

   $(c)\ \dfrac{x}{x-6}+1=\dfrac{6}{x-6}$ $\quad (d)\ \dfrac{5}{x-2}=\dfrac{-3}{x+2}+\dfrac{28}{(x-2)(x+2)}$

3. A total of \$51,000 is to be invested, some in bonds and some in certificates of deposit (CDs). If the amount invested in bonds is to exceed that in CDs by \$3,000, how much will be invested in each type of investment?

4. Shannon, who is paid time-and-a-half for hours worked in excess of 40 hours, had gross weekly wages of \$608 for 56 hours worked. What is her regular hourly wage?

Teaching Notes:

- Check their answers in the original equation.
- The necessity of noting the domain for rational equations.

Answers:

1. $(a)\ x=0$ $\quad (b)\ x=5$ $\quad (c)\ x=\dfrac{46}{5}$

2. $(a)\ x=16$ $\quad (b)\ x=6$ $\quad (c)\ \textit{No solution}$ $\quad (d)\ x=3$

3. \$24,000 in CDs; \$27,000 in bonds

4. \$9.50/hour

Mini-Lecture 1.2
Quadratic Equations

Learning Objectives:

1. Solve a quadratic equation by (a) factoring, (b) completing the square, (c) the quadratic formula
2. Solve problems that can be modeled by quadratic equations

Examples:

1. Find the real solutions by factoring: $3x^2+4x-4=0$.
2. Find the real solutions by using the square root method: $(4x-1)^2-16=0$.
3. Find the real solutions by completing the square: $x^2+4x-10=0$.
4. Find the real solutions by using the quadratic formula: $3x^2-5x-7=0$.
5. A ball is thrown vertically upward from the top of a building 48 feet tall with an initial velocity of 32 feet per second. The distance s (in feet) of the ball from the ground after t seconds is $s=48+32t-16t^2$.
   (a) After how many seconds does the ball strike the ground?
   (b) After how many seconds will the ball pass the top of the building on its way down?

Teaching Notes:

- Some that do not have good skills will struggle with this section. Most students can factor pretty well, but they will commit many types of algebraic mistakes when using the other methods.
- When you use the quadratic formula, you may have trouble simplifying the rational expression. For example, $\frac{10\pm 5\sqrt{10}}{10}=1\pm 5\sqrt{10}$ is a common error.
- Completing the square will shine a light on the difficulties that students have with fractions.

Answers:

1. $\frac{2}{3};-2$ 2. $\frac{5}{4};-\frac{3}{4}$ 3. $-2\pm\sqrt{14}$

4. $\frac{5\pm\sqrt{109}}{6}$ 5. (a) 3 seconds (b) 2 seconds

Mini-Lecture 1.3
Complex Numbers; Quadratic Equations in the Complex Number System

Learning Objectives:

1. Add, subtract, multiply, and divide complex numbers
2. Solve quadratic equations in the complex number system

Examples:

1. Write each expression in the standard form $a+bi$.
   $(a)\ (2-9i)+(9+7i)$ $\quad (b)(2-4i)-(5+2i)$ $\quad (c)\ (7-4i)(2+i)$
   $(d)\ \dfrac{4}{7-4i}$ $\quad (e)\ \dfrac{6-i}{7+i}$ $\quad (f)\ i^{18}$ $\quad (g)\ (1+i)^3$

2. Perform the indicated operation and express the answer in the form $a+bi$.
   $(a)\ \sqrt{-100}$ $\quad (b)\ \sqrt{(2+5i)(2-5i)}$

3. Solve each equation in the complex number system.
   $(a)\ x^2+5=0$ $\quad (b)\ x^2+2x+7=0$
   $(c)\ 2x^2-4x-5=0$ $\quad (d)\ x^2-2x+5=0$

Teaching Notes:

- If the you can use the quadratic formula, then you will not have a problem with this section.
- You need to know how to reduce radical and rational expressions.

Answers:

1. $(a)\ 11-2i$ $\quad (b)\ -3-6i$ $\quad (c)\ 18-i$ $\quad (d)\ \dfrac{28}{65}+\dfrac{16}{65}i$ $\quad (e)\ \dfrac{41}{50}-\dfrac{13}{50}i$ $\quad (f)\ -1$ $\quad (g)\ -2+2i$

2. $(a) 10i$ $\quad (b)\ \sqrt{49}$

3. $(a) x=\pm\sqrt{5}i$ $\quad (b)\ x=-1\pm\sqrt{6}i$ $\quad (c)\ x=\dfrac{2\pm\sqrt{14}}{2}$ $\quad (d)\ x=1\pm 2i$

Mini-Lecture 1.4
Radical Equations; Equations Quadratic in Form; Factorable Equations

Learning Objectives:

1. Solve radical equations
2. Solve equations quadratic in form
3. Solve equations by factoring

Examples:

1. Find the real solutions of each equation.

   $(a)\ \sqrt{2x-4}=4$ $\quad(b)\ \sqrt{7-6x}=x$ $\quad(c)\ x=2\sqrt{6x-36}$

   $(d)\ \sqrt{x^2-x-7}=x+3$ $\quad(e)\ \sqrt{3x+1}-\sqrt{x-1}=2$ $\quad(f)\ (3x+3)^{1/2}=9$

2. Find the real solutions of each equation.

   $(a)\ 14x^4-5x^2-1=0$ $\quad(b)\ (x+6)^2+3(x+6)+2=0$ $\quad(c)\ x+\sqrt{x}=30$

   $(d)\ \dfrac{1}{(x+6)^2}=\dfrac{1}{(x+6)}+12$ $\quad(e)\ 8x^{2/3}-39x^{1/3}-5=0$

3. Find the real solutions of each equation by factoring.

   $(a)\ x^3-49x=0$ $\quad(b)\ 7x^3=2x^2$ $\quad(c)\ x^3-14x^2+48x=0$ $\quad(d)\ x^3+x^2-25x-25=0$

Teaching Notes:

- Radical equations such as part d in example 1 above will result in such mistakes as $\sqrt{x^2-x-7}=x+3 \Rightarrow x^2-x-7=x^2+9$. Students will square both sides incorrectly. Make sure you go over this.
- The necessity of checking answers in radical equations.
- If you use substitution to solve problems such as those in example 2 above, make sure you reiterate the necessity of substituting back and continuing on to find the true solution. Again, make them check their answers.

Answers:

1. $(a)\ 10$ $\quad(b)\ 1$ $\quad(c)\ 12$ $\quad(d)\ -\dfrac{16}{7}$ $\quad(e)\ 1,5$ $\quad(f)\ 26$

2. $(a) -\dfrac{\sqrt{2}}{2},\dfrac{\sqrt{2}}{2}$ $\quad(b)\ -7,-8$ $\quad(c)\ 25$ $\quad(d)\ -\dfrac{19}{3},-\dfrac{23}{4}$ $\quad(e)\ -\dfrac{1}{512},125$

3. $(a)\ -7,0,7$ $\quad(b)\ 0,\dfrac{2}{7}$ $\quad(c)\ 0,6,8$ $\quad(d)\ -1,-5,5$

Mini-Lecture 1.5
Solving Inequalities

Learning Objectives:

1. Use interval notation
2. Use properties of inequalities
3. Solve inequalities
4. Solve combined inequalities

Examples:

1. Write each inequality using interval notation.

   $(a)\ -4 < x \le 5$ $\quad (b)\ 0 < x < 4$ $\quad (c)\ 5 \le x < 12$ $\quad (d)\ 8 \le x \le 16$

2. Write each interval as an inequality involving *x*.

   $(a)\ (-5,10)$ $\quad (b)\ [4,16]$ $\quad (c)\ [0,6)$ $\quad (d)\ (-5,1]$

3. Solve each inequality. Express the answer in interval notation.

   $(a)\, x+7<1$ $\quad (b)\, 3x-9 \ge 3+x$ $\quad (c)\ 8-7(1-x) \le 7$ $\quad (d)\ \dfrac{x}{4} \ge 3-\dfrac{x}{16}$

4. Solve each inequality. Express the answer in interval notation.

   $(a)\ -15 \le 9-4x \le 25$ $\quad (b)\ -4 < \dfrac{4x-8}{5} < 0$ $\quad (c)\ -2 < 1-\dfrac{1}{2}x < 5$

   $(d)\ (x+5)(x-7) > (x-5)(x+5)$ $\quad (e)\ (8x+4)^{-1} < 0$ $\quad (f)\ 0 < \dfrac{5}{x} < \dfrac{8}{9}$

Teaching Notes:

- The main problem that students will have is not reversing the inequality sign when multiplying or dividing by a negative.
- Interval notation. Some students have not seen this before and will need to understand it for many more topics.

Answers:

1. $(a)(-4,5]$ $\quad (b)\ (0,4)$ $\quad (c)\ [5,12)$ $\quad (d)\ [8,16]$
2. $(a)\ -5 < x < 10$ $\quad (b)\ 4 \le x \le 16$ $\quad (c)\ 0 \le x < 6$ $\quad (d)\ -5 < x \le 1$
3. $(a)\ (-\infty,-6)$ $\quad (b)\ [6,\infty)$ $\quad (c)\ \left(-\infty,\dfrac{6}{7}\right]$ $\quad (d)\ \left[\dfrac{48}{5},\infty\right)$
4. $(a)\ [-4,6]$ $\quad (b)\ (-3,2)$ $\quad (c)\ (-8,6)$ $\quad (d)\ (-\infty,-5)$ $\quad (e)\ \left(-\infty,-\dfrac{1}{2}\right)$ $\quad (f)\ \left(\dfrac{45}{8},\infty\right)$

Mini-Lecture 1.6
Equations and Inequalities Involving Absolute Value

Learning Objectives:

1. Solve equations involving absolute value
2. Solve inequalities involving absolute value

Examples:

1. Solve each equation.

   $(a)\ |5x-10|=15$ $\quad (b)\ \left|\frac{2}{3}x+6\right|=12$ $\quad (c)\ |4-3x|-4=1$ $\quad (d)\ |x^2+x-1|=1$

2. Solve each absolute value inequality.

   $(a)\ |3x|\le 21$ $\quad (b)\ |4x-3|\ge 9$ $\quad (c)\ |2-6x|-5<1$ $\quad (d)\ -|3x-3|\ge -8$

Teaching Notes:

- When solving absolute value equations, do not forget that there are two solutions.
- Isolate the absolute value expression before trying to solve, such as examples 1c and 2c above.
- Don't be surprised if you see an answer such as $-3<x>2$. One will invariably try to combine two intervals that cannot be combined.

Answers:

1. $(a)\ x\in\{-1,5\}$ $\quad (b)\ x\in\{-27,9\}$ $\quad (c)\ x\in\left\{-\frac{1}{3},3\right\}$ $\quad (d)\ x\in\{-2,-1,0,1\}$

2. $(a)\ [-7,7]$ $\quad (b)\ x\le -\frac{3}{2}\ or\ x\ge 3$ $\quad (c)\left(-\frac{2}{3},\frac{4}{3}\right)$ $\quad (d)\left[-\frac{5}{3},\frac{11}{3}\right]$

Mini-Lecture 1.7

Problem Solving: Interest, Mixture, Uniform Motion, and Constant Rate Job

Learning Objectives:

1. Translate verbal descriptions into mathematical expressions
2. Solve interest problems
3. Solve mixture problems
4. Solve uniform motion problems
5. Solve constant rate job problems

Examples:

1. Translate the following sentence into a mathematical equation.
   "The area, A, of a circle is the product of the number π and the square of the radius, r.
2. Betsy, a recent retiree, requires \$6,000 per year in extra income. She has \$70,000 to invest and can invest in B-rated bonds paying 17% per year or in a CD paying 7% per year. How much money should be invested in each to realize exactly \$6,000 in interest per year?
3. A nut store normally sells cashews for \$4 per pound and peanuts for \$1.50 per pound. At the end of the month the peanuts had not sold well, so, in order to sell 60 pounds of peanuts, the manager decided to mix the 60 pounds of peanuts with some cashews and sell the mixture for \$2.50 per pound. How many pounds of cashews should be mixed with the peanuts to ensure no change in the profit?
4. A boat can maintain a constant speed of 34 mph relative to the water. The boat makes a trip upstream to a certain point in 21 minutes; the return trip takes 13 minutes. What is the speed of the current?
5. Trent can deliver his newspapers in 60 minutes. It takes Lois 40 minutes to do the same route. How long would it take them to deliver the newspapers if they work together?

Answers:

1. $A = \pi r^2$ 2. \$11,000 at 17% and \$59,000 at 7% 3. 40 pounds of cashews

4. 8 mph 5. 24 minutes