Mini-Lecture 2.1
The Distance and Midpoint Formulas

Learning Objectives:

1. Use the Distance Formula
2. Use the Midpoint Formula

Examples:

1. Find the distance between the points (-3,7) and (4,10).

2. Determine whether the triangle formed by points A=(-2,2), B=(2,-1), and C=(5,4) form a right triangle.

3. Find the midpoint of the line segment joining the points P_1=(6,-3) and P_2=(4,2).

Teaching Notes:

- Go over the terms used in introducing the rectangular coordinate system.
- The distance formula will be used in several applications later in the course.
- You don't have much trouble with the distance formula, but you will sometimes reverse the order of the coordinates or will make careless arithmetic mistakes such as using subtraction instead of addition.
- The midpoint formula is also fairly easy for them, but you will sometimes have trouble if the coordinates include fractions.

Answers:

1. $\sqrt{58}$ 2. No: $|AB|^2 = 13,\ |BC|^2 = 34,\ |AC|^2 = 53.$ 3. $\left(5, -\frac{1}{2}\right)$

Mini-Lecture 2.2
Graphs of Equations in Two Variables; Intercepts; Symmetry

Learning Objectives:

1. Graph equations by plotting points
2. Find intercepts
3. Test for symmetry
4. Graph key equations

Examples:

1. Determine whether the points (0,3), (-2,0), and (2,7) are on the graph of the equation $y = x^3 - 2x + 3$.
2. Find the intercepts of the equation $y = 2x - 1$ by plotting points.

3. List the intercepts and test for symmetry for each equation.

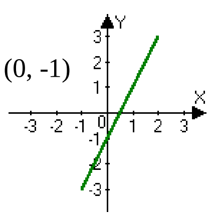
   $(a)\ y^2 - x - 4 = 0$ $\qquad (b)\ y = \dfrac{x}{x^2 - 4}$

4. Graph $x = y^2$

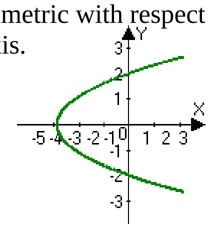
Teaching Notes:

- When graphing by plotting points, be sure to emphasize that the *y*-coordinate is determined by the value of *x*. This will help establish the function concept later.
- Emphasize how to find intercepts algebraically by setting *x=0* to find the *y*-intercept(s) and then *y=0*. to find the *x*-intercept(s).
- Symmetry can be seen and identified, but students will often have trouble testing for symmetry algebraically. They will make a lot of sign errors, so that needs to be reinforced.
- Emphasize the graphing of the key functions. It is important that they know the basic shapes of these graphs when this topic is revisited later in the course.

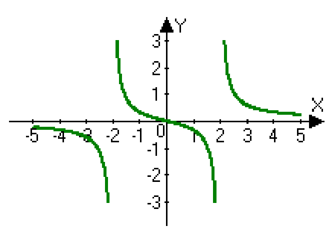
Answers: **1.** Yes, No, Yes **2.** (1/2,0), (0, -1)



3. (a) (0,2),(0,-2),(-4,0)
Symmetric with respect to the *x*-axis.



3. (b) (0,0) Symmetric with respect to the origin.



Mini-Lecture 2.3
Lines

<u>Learning Objectives</u>:

1. Calculate and interpret the slope of a line
2. Graph lines given a point and a slope
3. Find the equation of a vertical line
4. Use the point-slope form of a line; identify horizontal lines
5. Find the equation of a line, given two points
6. Write the equation of a line in slope-intercept form
7. Identify slope and *y*-intercept of a line from its equation
8. Graph lines written in general form using intercepts
9. Find equations of parallel lines
10. Find equations of perpendicular lines

<u>Examples</u>:

1. Determine the slope of the line containing the points (-5,4) and (0,7).
2. Graph the line containing the point $(2,4)$ with slope $m = \frac{-2}{3}$.
3. Write an equation of the line satisfying the given conditions:
   (a) Slope = $\frac{3}{4}$, containing the point (-2,4).   (b) Containing the points (4,2) and (3,-4).
   (c) *x*-intercept = 3, *y*-intercept = -2.   (d) Vertical line containing (5,-1).
   (e) Parallel to the line $3x - 4y = 5$ and containing the point (3,-6).
4. Find the slope and *y*-intercept of the line $4x - 6y = -3$.
5. Find the intercepts and graph the line $-2x + y = 4$.

<u>Teaching Notes</u>:

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- When finding the slope, make sure they don't reverse the *x*- and *y*-values.
- You should learn the various forms for the equation of a line and be comfortable solving standard form for *y*.
- Simplifying the equations should be emphasized.

Answers: 1. $m = \dfrac{7-4}{0-(-5)} = \dfrac{3}{5}$ 3. (a) $y = \frac{3}{4}x + \frac{11}{2}$

3. (b) $y = 6x - 22$ (c) $y = \frac{2}{3}x - 2$ (d) $x = 5$ (e) $y = \frac{3}{4}x - \frac{33}{4}$

4. Slope $= \dfrac{2}{3}$; *y*-intercept $= \dfrac{1}{2}$ 5. *x*-intercept = -2, *y*-intercept = 4

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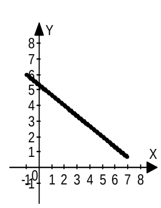

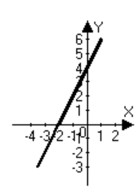

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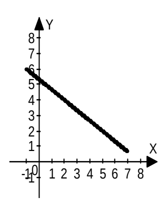
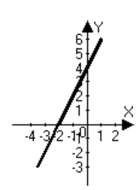




Mini-Lecture 2.4
Circles

Learning Objectives:

1. Write the standard form of the equation of a circle
2. Graph a circle
3. Work with the general form of the equation of a circle

Examples:

1. Write the standard form and general form of the equation of each circle with radius r and center (h,k). Graph each circle.

   $(a)\ r=3;\ (h,k)=(-2,3).\qquad (b)\ r=\frac{2}{3};\ (h,k)=(0,0).$

2. Find the center (h,k) and radius r of each circle.

   $(a)\ 2(x-2)^2+2(y+3)^2=8\qquad (b)\ x^2+y^2-6x+2y+4=0$

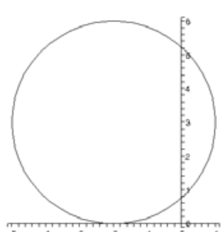
3. Find the general form of the equation of each circle.
   (a) Center (2,-3) and containing the point (0,4).
   (b) Endpoints of a diameter at (6,10) and (-4,-4).

Teaching Notes:

- Pay attention as how the difference in the equations of circles with centers at the origin and those with centers elsewhere.
- It is not necessary to memorize the general form of the equation.
- You will need review on completing the square (in order to put the general form into standard form).

Answers:

1. $(a)\ (x+2)^2+(y-3)^2=9;\ \ x^2+y^2+4x-6y+4=0.\quad (b)\ x^2+y^2=\frac{4}{9};\ \ x^2+y^2-\frac{4}{9}=0.$



2. $(a)\ c=(2,-3);\ r=2.\qquad (b)\ c=(3,-1);\ r=\sqrt{6}.$

3. $(a)\ (x-2)^2+(y+3)^2=53.\qquad (b)\ (x-1)^2+(y-3)^2=74.$

Mini-Lecture 2.5
Variation

Learning Objectives:

1. Construct a model using direct variation
2. Construct a model using inverse variation
3. Construct a model using joint or combined variation

Examples:

1. The monthly payment p on a mortgage varies directly with the amount borrowed B. If the monthly payment on a 30-year mortgage is $5.75 for every $1000 borrowed, find a function $p=p(B)$ that relates the monthly payment p to the amount borrowed B for a mortgage with the same terms. Then find the monthly payment p when the amount borrowed B is $225,000.

2. The length of a violin string varies inversely as the frequency of its vibrations. If a string 8 inches long vibrates at a frequency of 640 cycles per second, what is the frequency of a string that is 10 inches long?

3. The volume of a cone V varies jointly as its height h and the square of its radius r. A cone with a radius of 4 cm, and a height of 6 cm, has a volume of 32π. Find the volume of a cone having a radius of 15 cm and a height of 30 cm.

Teaching Notes:

- You may enjoy application problems, but these problems have a lot of interesting applications.
- Make sure you know how to distinguish between direct and inverse variation problems.

Answers:

1. $1293.75

2. 512 cycles per second

3. 2250π cm^3