Mini-Lecture 3.1
Functions

Learning Objectives:

1. Determine whether a relation represents a function
2. Find the value of a function
3. Find the domain of a function defined by an equation
4. Form the sum, difference, product, and quotient of two functions

Examples:

1. Determine whether the equation defines *y* as function of *x*.

$$(a)\ y = x^2 - 2x \quad (b)\ y^2 = 3x - 4 \quad (c)\ 5x + 7y = 10 \quad (d)\ y = \frac{2}{x-3}$$

2. For $f(x) = -x^2 + 2x - 3$ find $(a)\ f(0)$ $(b)\ f(-1)$ $(c)\ f(3)$
3. Find the domain of each function.

$$(a)\ f(x) = 2x + 3 \quad (b)\ f(x) = \frac{2}{x^2} \quad (c)\ f(x) = \frac{2x}{x^2+1} \quad (d)\ f(x) = \frac{5}{\sqrt{x+4}}$$

4. For $f(x) = 2x - 3$ and $g(x) = 2x^2$, find

$$(a)\ (f+g)(x) \quad (b)\ (f-g)(x) \quad (c)\ (f \cdot g)(2) \quad (d)\ \left(\frac{f}{g}\right)(3)$$

Teaching Notes:

- This is a critical section. If the you do not understand the concept of a function, you will struggle throughout the course.
- Spend time on the correspondence aspect of a function. You may use the birthday example. Every student has only one birthday, but other students can have that same birthday. Emphasize that no student has two birthdays.
- Demonstrate the difference between a relation and a function. A circle and a line are good geometric examples of this. A function is a special type of relation.
- The input – output machine in figure 10 is a good one to use extensively. Just keep emphasizing "input=domain, output=range".
- If time permits, introduce the difference quotient as a precursor to calculus limits.

Answers: 1. (a) yes (b) no (c) yes (d) yes 2. (a) -3 (b) -6 (c) -6

3. $(a)\ (-\infty,\infty)$ $(b)\ (-\infty,0)\cup(0,\infty)$ $(c)\ (-\infty,\infty)$ $(d)\ (-4,\infty)$

4. $(a)\ 2x^2 + 2x - 3$ $(b)\ -2x^2 + 2x - 3$ $(c)\ 8$ $(d)\ \frac{1}{6}$

Mini-Lecture 3.2
The Graph of a Function

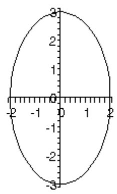
Learning Objectives:

1. Identify the graph of a function
2. Obtain information from or about the graph of a function

Examples:

1. Determine whether the graph is that of a function. If it is, then use the graph to find the domain, range, any intercepts, and symmetry with respect to the *x*-axis, the *y*-axis, or the origin.

(a)



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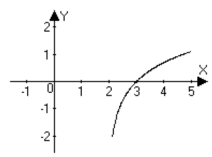


2. For $f(x)=\dfrac{2x}{x-2}$ answer the following questions.
   (a) Is the point (3,6) on the graph of *f*?
   (b) For $x = -2$, what is $f(x)$? What are the coordinates of that point on the graph $y=f(x)$?
   (c) If $f(x) = 3$, what is x?
   (d) What is the domain of *f*?
   (e) List any intercepts and zeros of *f*.

Teaching Notes:

- The vertical-line test is a useful tool if a student has a graph to analyze.
- Draw a lot of different graphs, and use the vertical-line test. This will help establish the concept of a function.
- Spend a good amount of time having the students read information from graphs. This is often something they have difficulty with. This will also help them later when they are learning about increasing and decreasing functions.

Answers:

1. (a) No (b) Function; Domain= $(2,\infty)$, Range = $(-\infty,\infty)$, *x*-int=3, no symmetry.

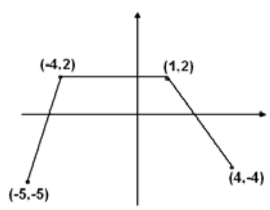
2. (a) Yes (b) 1; $(-2,1)$ (c) $x=6$ (d) $(-\infty,2)\cup(2,\infty)$

   (e) *x*-int=0; *y*-int=0; zero = 0.

Mini-Lecture 3.3
Properties of Functions

Learning Objectives:
1. Determine even and odd functions from a graph
2. Identify even and odd functions from the equation
3. Use a graph to determine where a function is increasing, decreasing, or constant
4. Use a graph to locate local maxima and local minima
5. Use a graph to locate the absolute maximum and the absolute minimum
6. Use a graphing utility to approximate local maxima and local minima and to determine where a function is increasing or decreasing
7. Find the average rate of change of a function

Examples:
1. For the graph below,
   (a) State the intervals where the function is increasing, decreasing, or constant.
   (b) State the domain and range.
   (c) State whether the graph is odd, even or neither.
   (d) Locate the maxima and minima.



2. Determine algebraically whether the function $f(x) = x^3 - 2x + 1$ is odd, even, or neither.
3. Find the average rate of change of $f(x) = -x^3 + 3x^2$ from x=-1 to x=4.

Teaching Notes:
- Graphically determining even and odd will not present a problem, but determining this algebraically can be difficult for students. Show the graph of a function and give its algebraic definition at the same time. This will help reinforce this concept.
- Determining increasing, decreasing, or constant is done fairly easily by just drawing some graphs and going over the properties. This is fairly intuitive. The main difficulty the students will have is showing the proper intervals. They will often want to use the *y*-values and not the *x*-values in their intervals.
- Students will usually give a point and not a value for a max or min. You need to understand that a max or min is a value and not an ordered pair.

Answers:
1. (a) Increasing on $(-5,-4)$; Decreasing on $(1,4)$; Continuous on $(-4,1)$
   (b) Domain = $[-5,4]$; Range = $[-5,2]$.   (c) Not odd or even.
   (d) Local Maximum = 2, Local Minimum = -5.
2. Neither   3. -4

Mini-Lecture 3.4
Library of Functions; Piecewise-defined Functions

Learning Objectives:

1. Graph the functions listed in the library of functions
2. Graph piecewise-defined functions

Examples: There are no variations from the library of functions in the exercises; this will be done in later sections. Therefore, these examples are of piece-wise functions only, but all of the library of functions are included in them.

1. Sketch the graph of each function.

$$(a)\ f(x)=\begin{cases} 2x-1 & \text{if } x>2,\\ 2-x & \text{if } x\le 2.\end{cases} \qquad (b)\ f(x)=\begin{cases} \dfrac{1}{x} & \text{if } x<0,\\ \sqrt{x} & \text{if } x\ge 0.\end{cases}$$

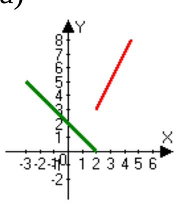
$$(c)\ f(x)=\begin{cases} x^3 & \text{if } x<1,\\ |x| & \text{if } x\ge 1.\end{cases} \qquad (d)\ f(x)=\begin{cases} x^2 & x<0,\\ 1 & x=0,\\ \sqrt[3]{x} & x>0.\end{cases}$$

Teaching Notes:

- It is a good idea to have the students memorize the graphs of the functions listed in the library of functions. Plotting points may help them at the beginning, but the graphs should be committed to memory.
- Students often have trouble graphing piecewise functions. If you can see the different parts in different colors, this can help you visualize the way the function is divided.

Answers:

1. (a)



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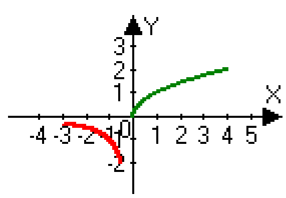

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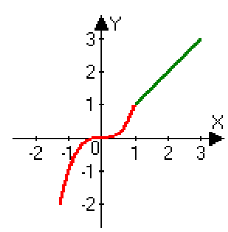

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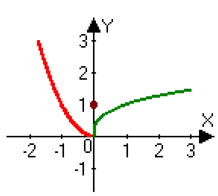


Mini-Lecture 3.5
Graphing Techniques: Transformations

Learning Objectives:

1. Graph functions using vertical and horizontal shifts
2. Graph functions using compressions and stretches
3. Graph functions using reflections about the *x*-axis and the *y*-axis

Examples:

1. Sketch the graph of each function.

$(a)\ f(x)=x^2-2$ $\quad (b)\ f(x)=x^3+3$ $\quad (c)\ f(x)=\sqrt{x+5}$ $\quad (d)\ f(x)=|x-2|$

$(e)\ f(x)=2x^2$ $\quad (f)\ f(x)=\frac{1}{2}x^3$ $\quad (g)\ f(x)=-\dfrac{1}{x}$ $\quad (h)\ f(x)=\sqrt{3-x}$

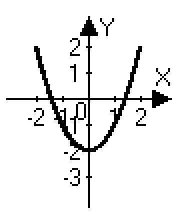
$(i)\ f(x)=(x-1)^2+2$ $\quad (j)\ f(x)=-\sqrt{2-x}+1$

Teaching Notes:

- If you can use a graphing calculator or computer projection with Mathematica or Maple, it is easy to show the transformations with multiple examples. The more examples the better.
- Using a table and plotting points can be helpful, but is too time-consuming to use on most graphs.
- Take just one of the functions, such as $f(x)=x^2$, and do all of the transformations. Then do the other functions.
- Problems like #63-66 are excellent ways to see if students understand the concept.

Answers:

(a)



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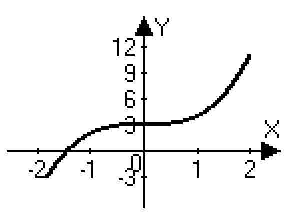

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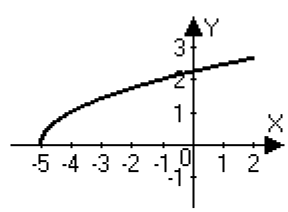

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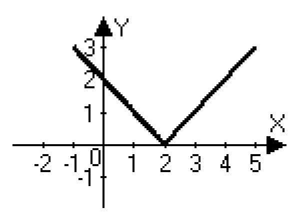

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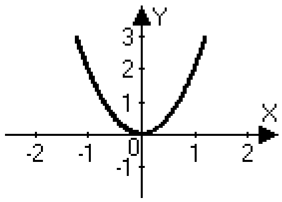
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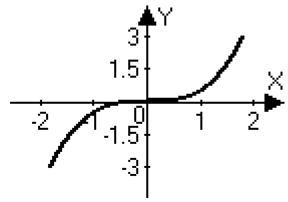
(f)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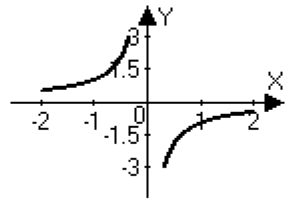
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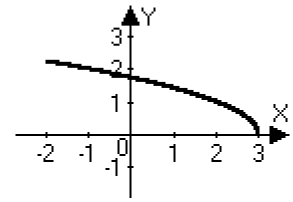
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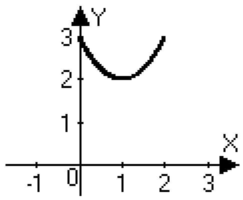
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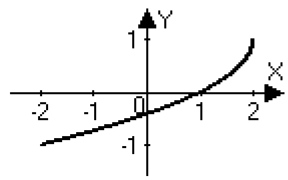
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Mini-Lecture 3.6
Mathematical Models: Building Functions

Learning Objectives:

1. Build and analyze functions

Examples:

1. Two cars are approaching an intersection. One is 1 mile south of the intersection and is moving at a constant speed of 40 mph. At the same time, the other car is 2 miles east of the intersection and is moving at a constant speed of 10 mph.
   (a) Express the distance *d* between the cars as a function of time *t*.
   (b) For what value of *t* is *d* smallest?

2. A rectangle has one corner on the graph of $y = 9 - x^2$, another at the origin, a third on the positive *y*-axis, and the fourth on the positive *x*-axis.
   (a) Express the area *A* as a function of *x*.
   (b) For what value of *x* is *A* the largest?
   (c) What is the domain of *A*?

3. Let $P = (x, y)$ be a point on the graph of $y = x^2 - 25$.
   (a) Express the distance *d* from *P* to the point $(1, 0)$ as a function of *x*.
   (b) What is *d* if *x*=2?

Teaching Notes:

- Try to use charts when possible.

Answers:

1. (a) $d = \sqrt{1700t^2 - 120t + 5}$ (b) $t \approx 0.035$ hours

2. (a) $A(x) = 9x - x^3$ (b) $x = \sqrt{3}$ (c) (0,3)

3. (a) $d = \sqrt{x^4 - 49x^2 - 2x + 626}$ (b) 21