Mini-Lecture 4.1
Linear Functions and Their Properties

Learning Objectives:
1. Graph linear functions
2. Use average rate of change to identify linear functions
3. Determine whether a linear function is increasing, decreasing, or constant
4. Build linear models from verbal description

Examples:
1. For each function, (i) determine the slope and *y*-intercept; (ii) graph the function using slope and *y*-intercept; (iii) determine the average rate of change; and (iv) determine whether the function is increasing, decreasing, or constant.

$(a)\ f(x)=3x+5.$ $(b)\ f(x)=-4x+2.$ $(c)\ f(x)=5.$

2. Suppose $f(x)=3x+6$ and $g(x)=-x+4$.

(a) Solve $f(x)=0$. (b) Solve $f(x)\geq 0$.

(c) Solve $f(x)=g(x)$. (d) Solve $f(x)\leq g(x)$.

3. The cost, *C*, in dollars of a cellular phone plan is given by the function $C(x)=0.30x+7$, where *x* is the number of minutes used.

(a) What is the cost of the plan if you talk for 150 minutes?
(b) If the bill is $220, how many minutes were used?
(c) What is the maximum number of minutes that can be used for $120?

Teaching Notes:
- The delta notation may take them a bit to understand.
- The applications are not difficult and should be interesting.

Answers: (Graphs are below.)
1. (a) (i) Slope = 3, *y*-intercept = 5; (iii) 3; (iv) increasing.
   (b) (i) Slope = -4, *y*-intercept = 2; (iii) -4; (iv) decreasing.
   (c) (i) Slope = 0, *y*-intercept = 5; (iii) 0; (iv) constant.
2. $(a)\ x=-2$ $(b)(-2,\infty)$ $(c)\ x=-\frac{1}{2}$ $(d)\left(-\infty,-\frac{1}{2}\right]$
3. (a) $52 (b) 710 (c) 376

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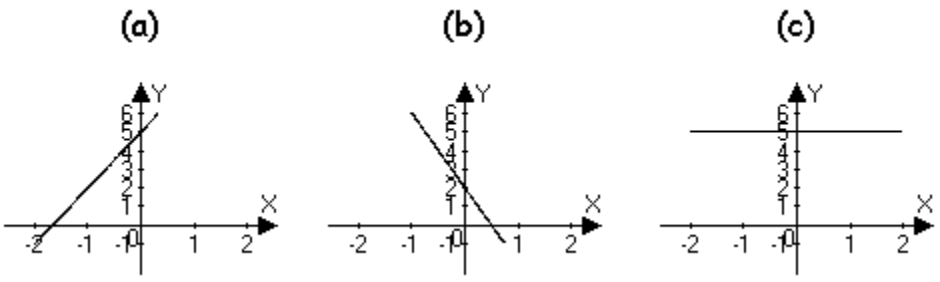


Mini-Lecture 4.2

Linear Models: Building Linear Functions from Data

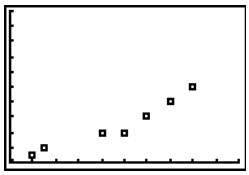
Learning Objectives:

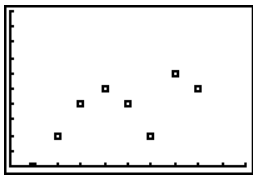
1. Draw and interpret scatter diagrams
2. Distinguish between linear and nonlinear relations
3. Use a graphing utility to find the line of best fit

Examples:

1. Examine the scatter diagram and determine whether the relation is linear or nonlinear.

(a) (b)



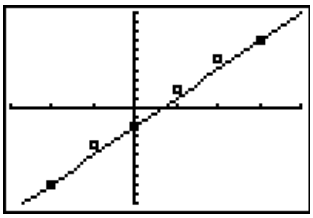


2. For the data below, draw a scatter diagram. Select two points from the diagram, and find the equation of the line containing the two points selected. Graph the line found on the scatter diagram.

x	-2	-1	0	1	2	3
y	-8	-4	-2	2	5	7

Answers:

1. (a) linear (b) nonlinear
2. $y = 3x - 2$



Mini-Lecture 4.3
Properties of Quadratic Functions

Learning Objectives:

1. Graph a quadratic function using transformations
2. Identify the vertex and axis of symmetry of a quadratic function
3. Graph a quadratic function using its vertex, axis, and intercepts
4. Find a quadratic function given its vertex and one other point
5. Find the maximum or minimum value of a quadratic function

Examples:

1. Graph each function by using transformations on the function $f(x) = x^2$.

$(a)\ f(x) = 2(x-2)^2 - 2$ $\quad$ $(b)\ f(x) = -3(x+1)^2 + 3$

2. Find the vertex, axis of symmetry, and intercepts, then graph the function. State the domain and range, where the function is increasing, and where it is decreasing.

$(a)\ f(x) = 2x^2 - 3x - 2$ $\quad$ $(b)\ f(x) = -x^2 - 4x$

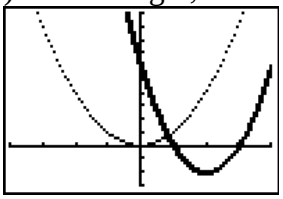
3. Find the quadratic function whose vertex is at (2,5) and passes through (3,2).
4. Determine the value of the maximum or the minimum without graphing.

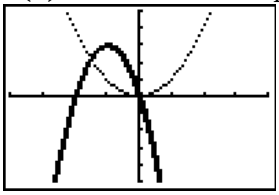
$(a)\ f(x) = 3x^2 - 24x + 53$ $\quad$ $(b)\ f(x) = -2x^2 - 12x - 24$

Teaching Notes:

- Initially, you may get confused with horizontal transformations, but vertical transformations don't cause too much difficulty.
- It is important that you learn to put the function in the form $f(x) = a(x-h)^2 + k$ in order to identify the vertex and to graph the function. It is also a good idea to use this form to find any *x*-intercepts by solving $a(x-h)^2 + k = 0$, especially when the function does not factor.
- It is important that you use the form $f(x) = ax^2 + bx + c$ to find the *y*-intercept. Otherwise they will think that the *k* value is the *y*-intercept.
- You may often state the vertex as the maximum or minimum instead of the *y*-value. Emphasize that the maximum or minimum is a value, not a coordinate.

Answers: 1.(a) Move 2 right, down 2, stretch of 2 (b) Reflect, 1 left, up 3, stretch of 3

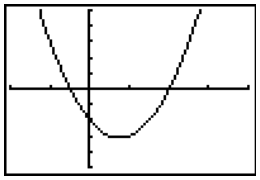




2. (a) Vertex = (3/4,-25/8); Axis of symmetry: x=3/4; *x*-intercepts -1/2, 2; *y*-intercept = -2
Domain = $(-\infty,\infty)$, Range = $[-25/8,\infty)$, Decreasing $(-\infty,3/4)$, Increasing $(3/4,\infty)$

(b) Vertex = (-2,4); Axis of symmetry: x=-2; *x*-intercepts -4, 0; *y*-intercept = 0
Domain = $(-\infty,\infty)$, Range = $(-\infty,4]$, Increasing $(-\infty,-2)$, Decreasing $(-2,\infty)$

(a)



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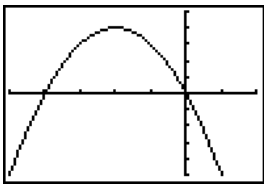

3. $f(x) = -3(x-2)^2 + 5$

4. (a) Minimum is 5 (b) Maximum is -6

Mini-Lecture 4.4
Build Quadratic Models from Verbal Descriptions and Data

Learning Objectives:

1. Build quadratic models from verbal descriptions
2. Build quadratic models from data

Examples:

1. An object is propelled straight upward from a height of 6 feet with an initial velocity of 32 feet per second. The height at any time *t* is given by $s(t) = -16t^2 + 32t + 6$ where $s(t)$ is measured in feet and *t* in seconds. Find the maximum height attained by the object.

2. A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals. What dimensions will produce a maximum enclosed area?

3. The revenue function for a new plasma television is given by $R(p) = 900p - 0.1p^2$. What price, *p*, should be charged to maximize revenue? What is the maximum revenue?

Answers:

1. 22 feet

2. $50\,\text{ft} \times 33\frac{1}{3}\,\text{ft}$

3. \$4500; \$2,025,000

Mini-Lecture 4.5
Inequalities Involving Quadratic Functions

Learning Objectives:

1. Solve inequalities involving a quadratic function

Examples:

1. Solve each inequality.

$(a)\ x^2 - x - 12 \le 0$

$(b)\ -2x^2 > -11x + 15$

$(c)\ 3x^2 + 6x > 45$

Teaching Notes:

- A problem that you may have with inequalities is the notation. Make sure you understand how to properly express the answer in interval notation.

Answers:

1. $(a)\ [-3, 4]$ $(b)\ \left(\frac{5}{2}, 3\right)$ $(c)\ (-\infty, -5) \cup (3, \infty)$