Mini-Lecture 5.1
Polynomial Functions and Models

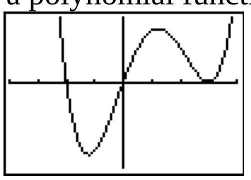
Learning Objectives:

1. Identify polynomial functions and their degree
2. Graph polynomial functions using transformations
3. Identify the real zeros of a polynomial function and their multiplicity
4. Analyze the graph of a polynomial function
5. Build cubic models from data

Examples:

1. Determine which functions are polynomials and state the degree. If not, state why.
   $(a)\ f(x)=5x^3-3x+4 \quad (b)\ f(x)=\sqrt{3}x^2 \quad (c)\ f(x)=\dfrac{3+x}{x-3} \quad (d)\ f(x)=\sqrt{x-3}$
2. Form a polynomial whose degree and zeros are given. Don't expand.
   (a) Degree 3; zeros: -2,0,4 (b) Degree 4; zeros: -2, multiplicity 3; 1, multiplicity 1

3. Find a polynomial function that could form the graph shown below.



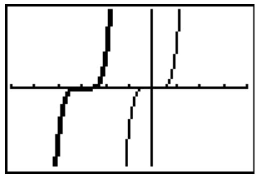
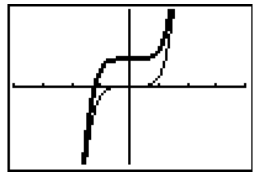
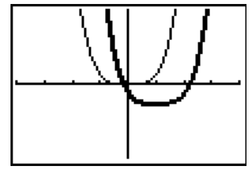
4. Use transformations of $y=x^4$ or $y=x^5$ to graph each function.
   $(a)\ f(x)=(x+3)^5 \quad (b)\ f(x)=x^5+3 \quad (c)\ f(x)=(x-1)^4-2$
5. Sketch the graph of each function by using end-behavior and multiplicity of zeros.
   $(a)\ f(x)=x^3(x-1)(x+3) \quad (b)\ f(x)=(x+2)^2(x-2)(x+4)$

Teaching Notes:

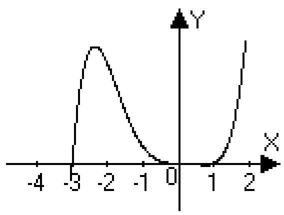
- It is essential to understand what constitutes a polynomial function.
- Understand the behavior of the functions at their zeros and the end behavior
- Giving students a generic graph using a,b,c instead of numerical values can be helpful in getting them to master the concepts.

Answers:

1. (a) Polynomial, degree 3 (b) Polynomial, degree 2 (c) Not a polynomial because the variable is in the denominator (d) Not a polynomial because of the radical
2. $(a)\ f(x)=x(x+2)(x-4) \quad (b)\ f(x)=(x+2)^3(x-1)$
3. $f(x)=x(x+2)(x-3)^2$

4. (a)  (b)  (c) 

5. (a)



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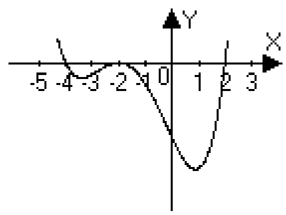


Mini-Lecture 5.2
Properties of Rational Functions

Learning Objectives:

1. Find the domain of a rational function
2. Find the vertical asymptotes of a rational function
3. Find the horizontal or oblique asymptotes of a rational function

Examples:

1. Find the domain of each rational function.

$(a)\ f(x)=\frac{2-x}{x+2}$ $(b)\ f(x)=\frac{3x^2}{x^2-9}$ $(c)\ f(x)=\frac{x-1}{x^2+1}$ $(d)\ f(x)=\frac{x}{x^2-4x-5}$

2. Identify vertical asymptotes, horizontal asymptotes, and oblique asymptotes.

$(a)\ f(x)=\frac{4x+1}{3-x}$ $(b)\ f(x)=\frac{2x^3+3x}{x^2-1}$ $(c)\ f(x)=\frac{x^4+1}{x^2-9}$ $(d)\ f(x)=\frac{2x}{x^2+1}$

3. Graph each function.

$(a)\ f(x)=\frac{1}{x}-2$ $(b)\ f(x)=\frac{2}{(x-2)^2}+1$

Teaching Notes:

- You may often remove from the domain values of *x* that make the numerator 0. For example, for $f(x)=\frac{x-3}{x^2-4}$, you will erroneously state the domain as $\{x|x\neq -2, x\neq 2, x\neq 3\}$.
- Analyzing the function to determine asymptotes so that you can learn to determine asymptotes quickly.

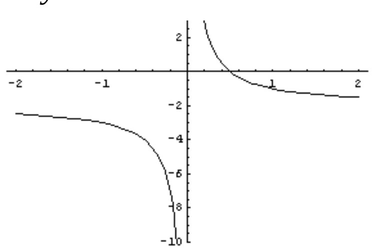
Answers:

1. $(a)\ \{x|x\neq -2\}$ $(b)\ \{x|x\neq -3, x\neq 3\}$ (c) All Real Numbers (d) $\{x|x\neq -1, x\neq 5\}$

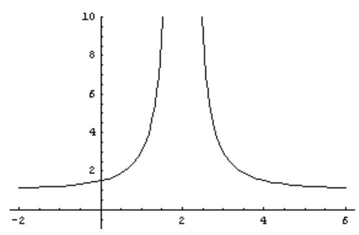
2. (a) V.A. $x=3$, H.A. $y=-4$ (b) Oblique $y=2x$, V.A. $x=1$, $x=-1$ (c) V.A. $x=3$, $x=-3$

   (d) H.A. $y=0$

3. (a)



(b)



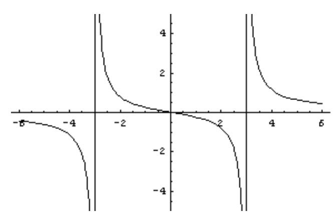
Mini-Lecture 5.3
The Graph of a Rational Function

Learning Objectives:
1. Analyze the graph of a rational function
2. Solve applied problems involving rational functions

Examples:
1. Analyze the graph of $f\left(x\right)=\dfrac{2x}{x^2-9}$ with the 7-step process used in Example 2.
2. A company that produces snowmobiles has a cost function, $C\left(x\right)=3500x+150,000$.
   (a) Find the average cost function. (b) What is the average cost of producing 100 snowmobiles?

Teaching Notes:
- You may usually do not enjoy application problems, but these problems have a lot of interesting applications.
- If you have a graphing calculator, try to do the graphs and use the calculator to check their work. You may need to grasp the concepts without too much dependency on a graphing calculator..
- Analyzing the functions without looking at the graph. Look at the graphs after the analysis is done, to check your work.

Answers:



1. Step 1: Domain is $\{x\,|\,x\neq 3, x\neq -3\}$, $f\left(0\right)=0$.
   Step 2: Reduced
   Step 3: VA are x = 3 and x = -3
   Step 4: HA is y = 0, intersects at the point (0,0)
   Step 5: On the interval $\left(-\infty,-3\right)$, the function is below the *x*-axis.
   On the interval $\left(-3,0\right)$, the function is above the *x*-axis.
   On the interval $\left(0,3\right)$, the function is below the *x*-axis.
   On the interval $\left(3,\infty\right)$, the function is above the *x*-axis.
   Step 6: As $x\to-\infty,\ f\left(x\right)\to 0;\ x\to-3^-,\ f\left(x\right)\to-\infty;\ x\to-3^+,\ f\left(x\right)\to\infty$
   $x\to\infty,\ f\left(x\right)\to 0;\ x\to 3^-,\ f\left(x\right)\to-\infty;\ x\to 3^+,\ f\left(x\right)\to\infty$
2. (a) $\overline{C}\left(x\right)=3500+\dfrac{150,000}{x}$ (b) \$5000

Mini-Lecture 5.4
Polynomial and Rational Inequalities

Learning Objectives:

1. Solve polynomial inequalities
2. Solve rational inequalities

Examples:

1. Solve each polynomial inequality.

$(a)\ (x+3)^2(x-4)>0$ $\qquad$ $(b)\ (x+1)(x-5)(x+3)\le 0$

$(c)\ x^6>16x^4$ $\qquad$ $(d)\ x^3+2x^2-8x\le 0$

2. Solve each rational inequality.

$(a)\ \dfrac{(x+3)^2}{x^2-4}\ge 0$ $\qquad$ $(b)\ \dfrac{2x-1}{x+4}\le 2$

$(c)\ \dfrac{x^2(x+3)(x-5)}{(x+2)(x-4)}\ge 0$ $\qquad$ $(d)\ \dfrac{2}{x-1}>\dfrac{3}{x+2}$

Teaching Notes:

- A sign chart can also be helpful in solving these inequalities.
- Using test numbers can be difficult if the roots are close together. You may not always calculate correctly.
- If using a graphing calculator in class, then this is a good section to use that technology in the presentation. Graphic representation gives you a clearer idea. The Table feature is a great way to calculate test values.

Answers:

1. $(a)\ (4,\infty)$ $\qquad$ $(b)\ (-\infty,-3]\cup[-1,5]$ $\qquad$ $(c)\ (-\infty,-4)\cup(4,\infty)$ $\qquad$ $(d)\ (-\infty,-4]\cup[0,2]$

2. $(a)\ (-\infty,-2)\cup(2,\infty)$ $\qquad$ $(b)\ (-4,\infty)$

$(c)\ (-\infty,-3]\cup(-2,4)\cup[5,\infty)$ $\qquad$ $(d)\ (-\infty,-2)\cup(1,7)$

Mini-Lecture 5.5
The Real Zeros of a Polynomial Function

Learning Objectives:

1. Use the Remainder and Factor Theorems
2. Use the Rational Zeros Theorem to list the potential rational zeros of a polynomial function
3. Find the real zeros of a polynomial function
4. Solve polynomial equations
5. Use the Theorem for Bounds on Zeros
6. Use the Intermediate Value Theorem

Examples:

1. Find the remainder if $f(x) = x^4 - 3x^3 + 2x - 4$ is divided by $(a)\ x-5 \quad (b)\ x+4$
2. Use the Remainder Theorem to determine whether the function $f(x) = 3x^4 - 6x^3 - 11x^2 + 4x + 6$ has the factor $(a)\ (x-3) \quad (b)\ (x+2)$.
3. Discuss the real zeros of $f(x) = 5x^5 - 3x^4 + 2x^3 + x^2 - 2x - 5$.
4. For the function $f(x) = 2x^4 + 5x^3 + x^2 + 10x - 6$, (a) list the potential rational zeros, (b) find the rational zeros.
5. Solve the equation $x^4 - 2x^3 - 8x^2 + 10x + 15 = 0$.
6. Find a bound to the zeros of $f(x) = x^5 - 4x^4 + 2x^3 - 5x + 2$.
7. Show that $f(x) = x^4 + x^3 - 9x^2 - 3x + 18$ has a root between -2 and -1.

Teaching Notes:

- This is a very important section for any student that will be taking a calculus course. Make sure you understand the Division Algorithm for Polynomials since this will form the basis for this section.
- Using a graphing calculator can speed up the process for the Rational Zeros Theorem.
- The Intermediate Value Theorem, as this is very important in calculus.

Answers:

1. (a) 256 (b) 436 2. (a) yes (b) no 3. Three or one positive real zeros. Two or none negative real zeros.

4. $(a)\ \frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$ $\quad (b)\ \frac{1}{2}, -3$ 5. $x = 3, -1, \pm\sqrt{5}$ 6. -6 and 6

7. $f(-1) = 12 > 0,\ f(-2) = -4 < 0$

Mini-Lecture 5.6
Complex Zeros; Fundamental Theorem of Algebra

Learning Objectives:

1. Use the Conjugate Pairs Theorem
2. Find a polynomial function with specified zeros
3. Find the complex zeros of a polynomial function

Examples:

1. Find the remaining two zeros of a polynomial of degree 6 whose coefficients are real numbers and has the zeros $2, -3, 2i,$ and $1-2i$.

2. Find a polynomial of degree 5 whose coefficients are real that has the zeros $0, -2i,$ and $2+i$.

3. Find the complex zeros of the polynomial function $f(x) = 2x^4 - 5x^3 - x^2 - 5x - 3$

Teaching Notes:

- This section brings all of the theorems learned about zeros together.
- See how the previous sections have pointed to this. Get them to see the "big picture", so to speak.
- A graphing calculator can really help to see the behavior of the polynomial.
- See how the graph of 4^{th} degree polynomial with only two real zeros, but there are 4 roots. This is a way to see the introduction of the complex zeros easy to be seen. A simple example is $x^4 - 16$.

Answers:

1. $-2i, 1+2i$

2. $f(x) = a(x^5 - 2x^4 + 9x^3 - 8x^2 + 20x)$

3. $i, -i, 3, -\frac{1}{2}$