# Mini-Lecture 6.1
## Composite Functions

**Learning Objectives:**

1. Form a composite function
2. Find the domain of a composite function

**Examples:**

1. For $f(x) = 2x+3$ and $g(x) = x^2 - 2x$, find
   $(a)\ (f \circ g)(2) \quad (b)\ (g \circ f)(2) \quad (c)\ (f \circ f)(-3) \quad (d)\ (g \circ g)(-1)$
2. Find the domain of $(f \circ g)(x)$ if $f(x) = \dfrac{2}{x-3}$ and $g(x) = \dfrac{1}{x+4}$.
3. Find the domain of $(f \circ g)(x)$ if $f(x) = \dfrac{1}{x+2}$ and $g(x) = \sqrt{x+1}$.
4. Show that $(f \circ g)(x) = (g \circ f)(x) = x$ for $(f)(x) = 3x-5$ and $g(x) = \dfrac{x+5}{3}$.
5. Find functions *f* and *g* such that $(f \circ g)(x) = H(x)$ for $H(x) = \sqrt{x+5}$.

**Teaching Notes:**

- The concept of a composite function is not difficult for most students, but the algebraic manipulations that are necessary will be problematic to some.
- You will struggle a little bit with the domain of a composite function if the functions are rational or radical functions. This will require some time and careful examples.
- Demonstrate that generally $(f \circ g)(x) \neq (g \circ f)(x)$, but that there are cases where $(f \circ g)(x) = (g \circ f)(x) = x.$. When this is true there is a special relationship between the two functions that will be studied in future sections.
- If time permits, we will go over the calculus application in Example 6, which is demonstrated in example 5 above. Emphasize the point that there is not a unique answer.

**Answers:**

1. $(a)\ 3 \quad (b)\ 35 \quad (c)\ -3 \quad (d)\ 3$
2. $\left\{x \mid x \neq -4, x \neq -\dfrac{11}{3}\right\}$
3. $\{x \mid x \geq -1\}$
4. $(f \circ g)(x) = (g \circ f)(x) = x.$
5. Answers can vary. One solution is $f(x) = \sqrt{x}$, $g(x) = x+5$.

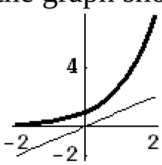
# Mini-Lecture 6.2
## One-to-One Functions; Inverse Functions

**Learning Objectives:**

1. Determine whether a function is one-to-one
2. Determine the inverse of a function defined by a map or a set of ordered pairs
3. Obtain the graph of the inverse function from the graph of the function
4. Find the inverse of a function defined by an equation.

**Examples:**

1. Determine whether the function $\{(2,-1),(3,2),(1,-1),(5,1)\}$ is one-to-one.
2. Find the inverse of the one-to-one function $\{(2,-1),(3,2),(1,3),(5,0)\}$.
3. For the graph shown, draw the graph of the inverse. The graph *y*=*x* is included to help.




4. For the function $f(x)=x^2-2,\ x\geq 0$, find $f^{-1}$, state the domain and range of both functions, and graph the functions as well as the line *y*=*x*.
5. Find the inverse of the function $f(x)=\dfrac{x}{2x+1}$.

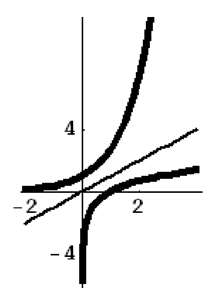
**Teaching Notes:**

- One of the most common mistakes made is that you think $f^{-1}(x)=\dfrac{1}{f(x)}$. The fact that the -1 is a notation and not an exponent must be made clear.
- It is important that you understand $D_f=R_{f^{-1}}$ and $D_{f^{-1}}=R_f$. This can be demonstrated graphically for emphasis.

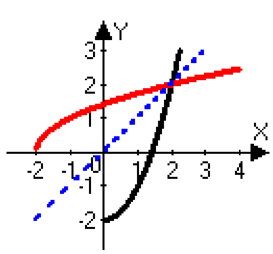
**Answers:**

1. No  2. $\{(-1,2),(2,3),(3,1),(0,5)\}$

3.




4. $f^{-1}(x)=\sqrt{x+2}$;  $D_f=R_{f^{-1}}=[0,\infty)$, $D_{f^{-1}}=R_f=[-2,\infty)$




5. $f^{-1}(x)=\dfrac{x}{1-2x}$

# Mini-Lecture 6.3
## Exponential Functions

**Learning Objectives:**

1. Evaluate exponential functions
2. Graph exponential functions
3. Define the number *e*
4. Solve exponential equations

**Examples:**

1. Evaluate: $(a)\ 3^{2.133}$ $(b)\ e^{-2.4}$ $(c)\ 4^{\sqrt{3}}$

2. Graph the following functions, and determine domain, range, and horizontal asymptote: $(a)\ f(x)=4^{x-2}-2$ $(b)\ f(x)=3-e^{-x}$.

3. Solve each equation. $(a)\ 2^{3x}=4^{x+1}$ $(b)\ e^{x^2}=e^{3x-2}$

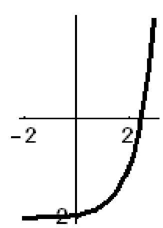
**Teaching Notes:**

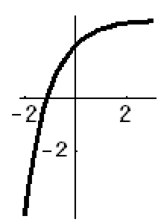
- Emphasize the basic shape of the graph of an exponential function and then do some simple transformations. This makes it an easy transition to the more difficult graphs.
- Use the horizontal asymptote and the restricted range, and mention that this will be revisited when the inverses of exponential functions are studied.
- Go over some of the multitude of real world applications of exponential functions (e.g., compound interest, growth, and decay).

**Answers:**

1. (a) 10.416 (b) 0.0907 (c) 11.0357

2. $(a)\ D=(-\infty,\infty),\ R=(-2,\infty),\ y=-2$ $(b)\ D=(-\infty,\infty),\ R=(-\infty,3),\ y=3$




3. $(a)\ x=2$ $(b)\ x=1,\ x=2$

# Mini-Lecture 6.4
## Logarithmic Functions

**Learning Objectives:**

1. Change exponential statements to logarithmic statements and logarithmic Statements to exponential statements
2. Evaluate logarithmic expressions
3. Determine the domain of a logarithmic function
4. Graph logarithmic functions
5. Solve logarithmic equations

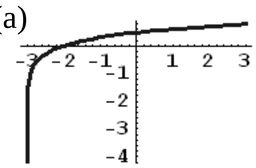
**Examples:**

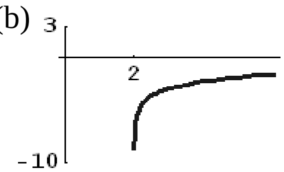
1. Rewrite each exponential expression as a logarithmic expression.
   $(a)\ 7^x = 15 \quad (b)\ \pi^3 = m \quad (c)\ x^5 = 12$
2. Rewrite each logarithmic expression as an exponential expression.
   $(a)\ \log_3 x = 4 \quad (b)\ \log_x 5 = 2 \quad (c)\ \log_3 8 = x$
3. Find the exact value of: $(a)\ \log_7 49 \quad (b)\ \log_5 \dfrac{1}{125} \quad (c)\ \ln e^4$
4. Find the domain of : $(a)\ f(x) = \log(2x-1) \quad (b)\ f(x) = \ln\left(\dfrac{x+1}{x-2}\right)$
5. Sketch the graph of : $(a)\ f(x) = \log(x+3) \quad (b)\ f(x) = -3 + \ln(x-2)$
6. Solve: $(a)\ \ln e^{2x} = 10 \quad (b)\ \log_4(x-2) = 2$

**Teaching Notes:**

- Emphasize the relationship between exponential and logarithmic expressions. You need to be proficient in changing from one form to another.
- Spend some time just evaluating logs and exponents on the calculator so that they can see the relationship that exists between them.
- Make sure you get in the habit of checking their answers in the equations. This will reinforce the definition.

**Answers:**

1. $(a)\ \log_7 15 = x \quad (b)\ \log_\pi m = 3 \quad (c)\ \log_x 12 = 5$
2. $(a)\ 3^4 = x \quad (b)\ x^2 = 5 \quad (c)\ 3^x = 8$ 3. $(a)\ 2 \quad (b)\ -3 \quad (c)\ 4$
4. $(a)\ D = \left(\dfrac{1}{2}, \infty\right) \quad (b)\ D = (-\infty, -1) \cup (2, \infty)$
5. (a)


(b)


6. (a) 5 (b) 18

# Mini-Lecture 6.5
## Properties of Logarithms

**Learning Objectives:**

1. Work with the properties of logarithms
2. Write a logarithmic expression as a sum or difference of logarithms
3. Write a logarithmic expression as a single logarithm
4. Evaluate logarithms whose base is neither 10 nor *e*

**Examples:**

1. Given $\ln 2 = a$, $\ln 3 = b$, and $\ln 5 = c$, use properties of logarithms to write in terms of a and b.

   $(a)\ \ln\frac{6}{5}$ $\qquad (b)\ \ln\sqrt[3]{25}$ $\qquad (c)\ \ln 24$

2. Write each logarithmic expression as a sum or difference of logarithms.

   $(a)\ \ln\left[\frac{2x\sqrt{x^2+1}}{x+3}\right]$ $\qquad (b)\ \log_2\left[\frac{x^2-4}{x+5}\right]$

3. Write each logarithmic expression as a single logarithm.

   $(a)\ \ln\left(x^2-25\right)-4\ln\left(x+5\right)$ $\qquad (b)\ 2\log\left(3x^3\right)-\frac{1}{3}\log\left(x^3-2\right)$

4. Use the Change-of-Base Formula to evaluate the following:

   $(a)\log_6 120$ $\qquad (b)\ \log_\pi 5.3$

**Teaching Notes:**

- Weaknesses in algebra will be amplified by this topic.
- Basic rules of factoring and reducing rational expressions.
- Some of you will need to be reminded of how to rewrite a radical expression as an exponential expression.
- These properties will be used when solving some logarithmic equations.

**Answers:**

1. $(a)\ a\cdot b-c$ $\qquad (b)\ \frac{2}{3}c$ $\qquad (c)\ 3a+b$
2. $(a)\ \ln 2x+\frac{1}{2}\ln\left(x^2+1\right)-\ln\left(x+3\right)$ $\qquad (b)\ \log_2\left(x-2\right)+\log_2\left(x+2\right)-\log_2\left(x+5\right)$
3. $(a)\ \ln\frac{(x-5)}{(x+5)^3}$ $\qquad (b)\ \log\frac{9x^6}{\sqrt[3]{x^3-2}}$
4. (a) 2.672 $\qquad$ (b) 1.457

# Mini-Lecture 6.6
## Logarithmic and Exponential Equations

**Learning Objectives:**

1. Solve logarithmic equations
2. Solve exponential equations
3. Solve logarithmic and exponential equations using a graphing utility

**Examples:**

1. Solve each equation.
   $(a)\ \log_3(x-5)+\log_3(x+3)=2$ $\quad(b)\ \log(3x-3)-\log 4=\log(x+1)$
   $(c)\ \log(x+4)=\log(5x+1)+\log 2$

2. Solve each equation.
   $(a)\ 3^{x+1}=7$ $\quad(b)\ e^{2x}-3e^{x}-4=0$ $\quad(c)\ 2^{x+2}=6^{2x-5}$

**Teaching Notes:**

- The need to check answers in logarithmic equations.
- If you have a graphing calculator, graphing the equations and finding the solution using an intersection is a good way to check the work.
- This is traditionally a fairly difficult topic. Their algebra weaknesses are magnified, so plan on going slow at first.

**Answers:**

1. $(a)\ 6$ $\quad(b)\ \varnothing$ $\quad(c)\ \dfrac{2}{9}$
2. $(a)\ x=\dfrac{\ln 7}{\ln 3}-1\approx 0.77124$ $\quad(b)\ x=\ln 4\approx 1.386$ $\quad(c)\ x=\dfrac{\ln 4+5\ln 6}{\ln 36-\ln 2}\approx 3.5792$

# Mini-Lecture 6.7
## Financial Models

**Learning Objectives:**

1. Determine the future value of a lump sum of money
2. Calculate effective rates of return
3. Determine the present value of a lump sum of money
4. Determine the rate of interest or time required to double a lump sum of money

**Examples:**

1. Find the amount that results from each investment.
   (a) $300 invested at 7.5% for 4 years, compounded quarterly
   (b) $500 invested at 6.7% for 6 years, compounded continuously

2. On January 2, 2007, $3000 is placed in an IRA that will pay interest of 7.5% per annum compounded continuously.
   (a) What will the IRA be worth on January 1, 2022?
   (b) What is the effective annual rate of interest?

3. Find the present value of:
   (a) $200 after 3 years at 5% compounded daily
   (b) $1000 after 5 years at 7% compounded continuously

4. How long does it take for money to double at 4.5%
   (a) compounded quarterly? (b) continuously?

**Teaching Notes:**

- This topic offers a great opportunity to show you a real world application that you will actually use in their life, and may already be using.
- This application gives you a good reason for learning how to solve exponential equations.
- There are usually ads in the newspaper that can be used to introduce and reinforce this topic.

**Answers:**

1. (a) $403.83 (b) $747.41

2. (a) $9240.65 (b) $7.788%

3. (a) $172.14 (b) $704.69

4. (a) 15.49 years (b) 15.4 years

# Mini-Lecture 6.8
## Exponential Growth and Decay Models;
## Newton's Law; Logistic Growth and Decay Models

**Learning Objectives:**

1. Find equations of populations that obey the law of uninhibited growth
2. Find equations of populations that obey the law of decay
3. Use Newton's Law of Cooling
4. Use logistic models

**Examples:**

1. A colony of bacteria increases according to the law of uninhibited growth.
   (a) If the number of bacteria doubles in 5 hours, find the function that gives the number of cells in the culture.
   (b) If there are 10,000 cells initially, how long will it take for there to be 25,000?
2. A bone is found to have 45% of the original amount of carbon-14. If the half-life of carbon-14 is 5,600 years, how old is the bone?
3. An object is heated to 90°C and is then allowed to cool in a room whose air temperature is 25°C. If the temperature of the object is 75°C after 10 minutes, when will its temperature be 50°C?
4. The function $P(t)=\dfrac{50,000}{1+60e^{-0.9t}}$ describes the number of people who have become ill with a virus $t$ weeks after its initial outbreak in a city of 50,000 inhabitants.
   (a) How many people were initially ill with the virus?
   (b) How many people were ill by the end of the 3rd week?

**Teaching Notes:**

- This section is another great opportunity to see some actual real world applications of mathematics.
- See how much mathematics is used in many fields, not just mathematics.
- There may be some confusion using Newton's Law of Cooling, so take the time to understand it carefully.

**Answers:**

1. $(a)\ N(t)=N_0e^{((\ln 2)/5)t}$ $\quad (b)\,t=\dfrac{\ln 5/2}{(\ln 2)/5}\approx 6.61$ hours

2. 6451 years

3. 36.4 minutes

4. (a) 819 (b) 9,935

# Mini-Lecture 6.9
## Building Exponential, Logarithmic, and Logistic Models from Data

**Learning Objectives:**

1. Build an exponential model from data
2. Build a logarithmic model from data
3. Build a logistic model from data

**Examples:**

1. Examine the following data.
   (a) Using a graphing utility, graph the data to determine if the relation best fits an exponential, logarithmic, or logistic model, then create a model from the data.
   (b) Use the function found in part (a) to predict the relation's value for $x = 30$.

| $x$ | $y$ |
|---|---|
| 2 | 80 |
| 4 | 65 |
| 8 | 45 |
| 10 | 40 |
| 12 | 35 |
| 20 | 22 |

**Teaching Notes:**

- You will have trouble choosing which model to use. Emphasize examining domain and range to distinguish exponential from logarithmic growth. Domain will identify vertical asymptotes (logarithmic model), and range will identify horizontal asymptotes (exponential model).

**Answers:**

1. (a) logarithmic; $y = 98.7 - 25.6\ln x$. (b) 11.6