Mini-Lecture 7.2
The Parabola

Learning Objectives:

1. Analyze parabolas with vertex at the origin
2. Analyze parabola with vertex at *(h,k)*
3. Solve applied problems involving parabolas

Examples:

1. Find the equation of the parabola described. Find the two points that define the latus rectum.
   (a) Focus at (7,0), vertex at (0,0)
   (b) Focus at (0,-1), directrix the line $y=1$.
   (c) Directrix $x=-\frac{1}{7}$, vertex at (0,0)
   (d) Vertex at (0,0), axis of symmetry the y-axis, contains the point (5,3)

2. Find the vertex, focus, and directrix of each parabola.
   $(a)\ y^2=4x$ $\quad (b)\ (x-5)^2=-(y+1)$
   $(c)\ (y-3)^2=12(x+1)$ $\quad (d)\ y^2-10y+4x+25=0$

3. A searchlight is shaped like a paraboloid of revolution. If the light source is located 2 feet from the base along the axis of symmetry and the opening is 10 feet across, how deep should the searchlight be?

Teaching Notes:

- There are numerous applications for the parabola. These should be emphasized.
- Take the time to go over all the terminology.
- There are numerous web sites that have applets demonstrating the properties.
- See the importance of learning the information in Table 1 and Table 2.

Answers:

1. $(a)\ y^2=28x;\ (7,14),(7,-14)$ $\quad (b)\ x^2=-4y;(2,-1),(-2,-1)$
   $(c)\ y^2=\frac{4}{7}x;\left(\frac{1}{7},\frac{2}{7}\right),\left(\frac{1}{7},-\frac{2}{7}\right)$ $\quad (d)\ x^2=\frac{25}{3}y;\left(-\frac{25}{6},\frac{25}{12}\right),\left(\frac{25}{6},\frac{25}{12}\right)$
2. $(a)\ V=(0,0)\ F=(1,0)\ x=-1$ $\quad (b)\ V=(5,-1)\ F\left(5,-\frac{5}{4}\right)\ y=-\frac{3}{4}$
   $(c)\ V=(-1,3)\ F=(2,3)\ x=-4$ $\quad (d)\ V=(0,5)\ F(-1,5)\ x=1$
3. 25/8

Mini-Lecture 7.3
The Ellipse

Learning Objectives:

1. Analyze ellipses with center at the origin
2. Analyze ellipses with center at *(h,k)*
3. Solve applied problems involving ellipses

Examples:

1. Find the vertices and foci of each ellipse. $(a)\ \frac{x^2}{64}+\frac{y^2}{36}=1$ $(b)\ 9x^2+y^2=81$
2. Find an equation for each ellipse.
   (a) Center = (0,0), Focus at (4,0), Vertex at (5,0)
   (b) Foci at (0,4) and (0,-4), length of major axis is 14

3. Find the center, foci, and vertices of each ellipse.
   $(a)\ \frac{(x-1)^2}{36}+\frac{(y+2)^2}{9}=1$ $(b)\ x^2-4x+16y^2+96y+4=0$
4. Find an equation for each ellipse.
   (a) Center =(-3,1), Vertex at (-3,10), Focus at (-3,6)
   (b) Foci at (-1,2), (-1,-4) and length of major axis is 14.

5. A hall 120 feet in length is to be designed as a whispering gallery. If the foci are located 35 feet from the center, how high is will the ceiling be at the center?

Teaching Notes:

- See the importance of learning the information in Table 3.
- Spend time on applications since there are many. The Whispering Gallery in example 7 is a good one to go over.
- Kepler's Law of Planetary Motion is demonstrated on http://home.cvc.org/science/kepler.htm which is a great way to introduce ellipses.

Answers:

1. $(a)\ V=(\pm 8,0),\ F=\left(\pm 2\sqrt{7},0\right)$ $(b)\ V=(0,\pm 9),\ F=\left(0,\pm 6\sqrt{2}\right)$
2. $(a)\ \frac{x^2}{25}+\frac{y^2}{9}=1$ $(b)\ \frac{x^2}{33}+\frac{y^2}{49}=1$
3. $(a)\ C=(1,-2),\ V=(7,-2)\,\&\,(-5,-2),\ F=\left(1\pm 3\sqrt{3},-2\right)$
   $(b)\ C=(2,-3),\ V=(14,-3)\,\&\,(-10,-3),\ F=\left(2\pm 3\sqrt{15},-3\right)$
4. $(a)\ \frac{(x+3)^2}{56}+\frac{(y-1)^2}{81}=1$ $(b)\ \frac{(x+1)^2}{40}+\frac{(y+1)^2}{49}=1$
5. 48.7 feet

Mini-Lecture 7.4
The Hyperbola

Learning Objectives:

1. Analyze hyperbolas with center at the origin
2. Find the asymptotes of a hyperbola
3. Analyze hyperbolas with center at *(h,k)*
4. Solve applied problems involving hyperbolas

Examples:

1. Find an equation for the hyperbola described.
   (a) Center (0,0), focus (0,8), vertex (0,2)
   (b) Foci $(0,\pm 25)$, vertex (0,24)
   (c) Foci $(\pm 6, 0)$, asymptote $y=-x$
2. Find the center, transverse axis, vertices, foci, and asymptotes.
   $(a)\ \frac{y^2}{81}-\frac{x^2}{16}=1 \qquad (b)\ 9x^2-y^2=81$
3. Find an equation for the hyperbola described.
   (a) Center $(4,-2)$, focus $(9,-2)$, vertex $(6,-2)$
   (b) Foci (4,10) and (10,10), vertex (9,10)
   (c) Vertices $(-1,-1)$ and $(5,-1)$, asymptote $y+1=\frac{5}{3}(x-2)$
4. Find the center, transverse axis, vertices, foci, and asymptotes.
   $(a)\ \frac{(y+4)^2}{36}-\frac{(x-3)^2}{4}=1 \qquad (b)\ x^2-y^2-4x-10y-25=0$

Teaching Notes:

- Hyperbolas have many interesting applications that should be emphasized.
- See the importance of learning the information in Table 4.
- It is important to be accurate and clear when graphing. Use colors if possible.

Answers:

1. $(a)\ \frac{y^2}{4}-\frac{x^2}{60}=1 \qquad (b)\ \frac{y^2}{576}-\frac{x^2}{49}=1 \qquad (c)\ \frac{x^2}{18}-\frac{y^2}{18}=1$

2. $(a)\ (0,0);\ y-\text{axis};\ (0,\pm 9);\ \left(0,\pm\sqrt{97}\right);\ y=\pm\frac{9}{4}x \qquad (b)\ (0,0);\ y-\text{axis};\ (\pm 3,0);\ \left(\pm 3\sqrt{10},0\right);\ y=\pm 3x$

3. $(a)\ \frac{(x-4)^2}{4}-\frac{(y+2)^2}{21}=1 \qquad (b)\ \frac{(x-7)^2}{4}-\frac{(y-10)^2}{5}=1 \qquad (c)\ \frac{(x-2)^2}{9}-\frac{(y+1)^2}{25}=1$

4. $(a)(3,-4);\ y-\text{axis};\ (3,-10)\ \&\ (3,2);\ \left(3,-4\pm 2\sqrt{10}\right);\ y+4=\pm 3(x-3)$
   $(b)(2,-5);\ x-\text{axis};\ (0,-5)\ \&\ (4,-5);\ \left(2\pm 2\sqrt{2},-5\right);\ y+5=\pm(x-2)$