Mini-Lecture 9.1
Sequences

Learning Objectives:

1. Write the first several terms of a sequence
2. Write the terms of a sequence defined by a recursive formula
3. Use summation notation
4. Find the sum of a sequence

Examples:

1. Write down the first five terms of the sequence.

$$(a)\ \left\{\frac{n}{n+9}\right\} \qquad (b)\ \left\{\frac{3^n}{4^n+1}\right\} \qquad (c)\ \left\{\frac{(-1)^n}{(n+3)(n+6)}\right\}$$

2. Write down the nth term of the sequence suggested by the pattern $1, \frac{1}{6}, \frac{1}{36}, \frac{1}{216}, \cdots$

3. Write the first five terms of the sequence defined by the recursive formula.

$$(a)\ a_1 = 1;\ a_n = 4a_{n-1} \qquad (b)\ a_1 = 2;\ a_n = \frac{a_{n-1}}{n^2} \qquad (c)\ a_1 = -4;\ a_n = n - a_{n-1}$$

4. Find the sum of the sequence.

$$(a)\ \sum_{k=1}^{20} 3 \qquad (b)\ \sum_{k=1}^{5} 4k+9 \qquad (c)\ \sum_{k=1}^{5} (-1)^k\, 4^k \qquad (d)\ \sum_{k=1}^{5} \left(k^3+3\right)$$

Teaching Notes:

- This will be totally new material for many students so it is important that sufficient time be spent establishing the terminology.
- Factorial notation is used in calculus, so stress this.

Answers:

1. $(a)\ \frac{1}{10}, \frac{2}{11}, \frac{1}{4}, \frac{4}{13}, \frac{5}{14} \qquad (b)\ \frac{3}{5}, \frac{9}{17}, \frac{27}{65}, \frac{81}{257}, \frac{243}{1025} \qquad (c)\ \frac{-1}{28}, \frac{1}{40}, \frac{-1}{54}, \frac{1}{70}, \frac{-1}{88}$

2. $\frac{1}{6^{n-1}}$

3. $(a)\ 1, 4, 16, 64, 256 \qquad (b)\ 2, \frac{1}{2}, \frac{1}{18}, \frac{1}{288}, \frac{1}{7200} \qquad (c)\ -4, 6, -3, 7, -2$

4. (a) 60 (b) 105 (c) -820 (d) 240

Mini-Lecture 9.2
Arithmetic Sequences

Learning Objectives:

1. Determine if a sequence is arithmetic
2. Find a formula for an arithmetic sequence
3. Find the sum of an arithmetic sequence

Examples:

1. For each arithmetic sequence, find the common difference, and write out the first four terms. $(a)\ \{2n+8\}$ $(b)\ \{\ln 4^n\}$
2. Find the *n*th term of the arithmetic sequence, and then find the fifth term if the initial term is $a=5$ and the common difference is $d=8$.
3. Find the indicated term for the given arithmetic sequence.
   (a) 11^{th} term of 7,14,21,… (b) 7^{th} term of -2, -7, -12,…
4. Find the first term and the common difference for the arithmetic sequence described. Give a recursive formula for the sequence.
   (a) 5^{th} term is 4; 17^{th} term is 28
   (b) 8^{th} term is -4; 19^{th} term is 51
5. Find the sum. $-15+(-24)+(-33)+\ldots+(-6-9n)$

Teaching Notes:

- Sequences are used in calculus, so it is important that you understand the terminology.
- This topic is a good opportunity to see the really interesting applications of mathematics.
- Stress that a sequence is a list of terms, not a sum.

Answers:

1. (a) Common difference = 2, First 4 terms: 10, 12, 14, 16
   (b) Common difference = 1.386 First 4 terms: 1.386, 2.772, 2.158, 5.544

2. $a_n = 5+(n-1)(8)$

3. $(a)\ a_{11} = 77$ $(b)\ a_7 = -32$

4. (a) First term is -4. Common difference is 2. $a_1 = -4;\ a_n = a_{n-1}+2$
   (b) First term is -39. Common difference is 5. $a_1 = -39;\ a_n = a_{n-1}+5$

5. $s_n = \frac{n}{2}(-21-9n)$

Mini-Lecture 9.3
Geometric Sequences; Geometric Series

Learning Objectives:

1. Determine if a sequence is geometric
2. Find a formula for a geometric sequence
3. Find the sum of a geometric sequence
4. Determine whether a geometric series converges or diverges
5. Solve annuity problems

Examples:

1. Determine whether the given sequence is arithmetic (find the common difference), geometric (find the common ratio), or neither.

   $(a)\ \{1,5,10,16,\ldots\}\quad (b)\ \left\{\left(\frac{3}{5}\right)^n\right\}$

2. Find a formula for a geometric sequence with $a=-3$ and $r=2$.
3. Find the sum of the geometric sequence $\frac{1}{6}+\frac{5}{6}+\frac{5^2}{6}+\frac{5^3}{6}+\cdots+\frac{5^{n-1}}{6}$.
4. Find the sum of the geometric series $4-\frac{1}{4}+\frac{1}{64}-\frac{1}{1024}+\cdots$
5. Determine whether each infinite geometric series converges or diverges. Find the sum if it converges.

   $(a)\ \sum_{k=1}^{\infty}9\left(-\frac{1}{2}\right)^{k-1}\qquad (b)\ \sum_{k=1}^{\infty}2\left(\frac{5}{3}\right)^{k-1}\qquad (c)\ \sum_{k=1}^{\infty}3\left(\frac{4}{5}\right)^{k-1}$

6. Arnold contributes $200 at the end of each quarter to a Tax Sheltered Annuity. What will the value of the TSA be after the 80$^{\text{th}}$ deposit (20 years) if the per annum rate of return is assumed to be 9% compounded quarterly?

Teaching Notes:

- See the difference between a sequence and a series.
- Geometric sequences are used in many convergence tests.
- Sequences and series are used extensively in calculus so students need to understand these concepts.
- Sometimes students will have trouble determining the value of *r* when they are testing a geometric series for convergence. Tell them they can divide the 2$^{\text{nd}}$ term by the first term and that will give them the value of *r*.

Answers:

1. (a) Neither (b) Geometric; common ratio is 3/5.

2. $a_n=-3(2)^{n-1}$ 3. $s_n=\frac{1}{24}\left(5^n-1\right)$ 4. $\frac{64}{17}$

5. (a) 6 (b) Divergent (c) 15 6. $43,823.5

Mini-Lecture 9.4
Mathematical Induction

Learning Objectives:

1. Prove statements using mathematical induction

Examples:

1. Use the principle of mathematical induction to show that the following statement is true for all natural numbers n.

$$18 + 36 + 54 + \ldots + 18n = 9n(n+1)$$

Teaching Notes:

- See the usefulness of learning to think analytically.

Answers:

1. Show the statement holds true for n=1.
   Assume the statement holds for some k, and determine w

Mini-Lecture 9.5
The Binomial Theorem

Learning Objectives:

1. Evaluate $\binom{n}{j}$
2. Use the Binomial Theorem

Examples:

1. Evaluate each expression.

   $(a)\binom{9}{6}\quad (b)\binom{6}{3}\quad (c)\binom{62}{61}$

2. Expand the expression using the Binomial Theorem.

   $(a)\ (x-3)^6\qquad (b)\ (3x+1)^4$

3. Use the Binomial Theorem to find the indicated coefficient or term.

   (a) Coefficient of x^5 in the expansion of $(x+2)^9$.

   (b) Coefficient of x^4 in the expansion of $(3x+1)^{12}$.

   (c) Third term in the expansion of $(x-3)^7$.

   (d) Coefficient of x^0 in the expansion of $\left(x^2-\frac{1}{x}\right)^{18}$.

Teaching Notes:

- This will be useful for any of you taking higher level calculus courses or any course using combinations.
- Show how to use Pascal's Triangle.

Answers:

1. (a) 84 (b) 20 (c) 62

2. $(a)\ x^6-18x^5+135x^4-540x^3+1215x^2-1458x+729$

   $(b)\ 81x^4+108x^3+54x^2+12x+1$

3. (a) 2016 (b) 40,095 (c) $189x^5$ (d) 18564