Mini-Lecture R.2
Algebra Essentials

Learning Objectives:

1. Graph inequalities
2. Find distance on the real number line
3. Evaluate algebraic expressions
4. Determine the domain of a variable
5. Use the laws of exponents
6. Evaluate square roots
7. Use scientific notation

Examples:

1. Replace the question mark by <, >, or =. $(a)\ \frac{8}{9}?0.89 \quad (b)\ \frac{5}{6}?0.83$
2. Graph $x \geq -3$ on the number line.
3. On the real number line, what is the distance between -5 and 2?
4. Evaluate if $x=-6$ and $y=2$: $(a)\ \frac{4x+3y}{6+6y} \quad (b)\ |2x-4y|$
5. Determine the value(s) of x that must be excluded from the domain of the variable in $\frac{x^2+8x-3}{x^3-4x}$.
6. Simplify each expression:

   $(a)\ 10^{-2} \quad (b)\ 8^{-3}\cdot 8 \quad (c)\ \sqrt{(-4)^2} \quad (d)\ \left(x^9y^{-4}\right)^6 \quad (e)\ \left(\frac{4x^{-5}}{7y^{-8}}\right)^{-3}$
7. Write in scientific notation: $(a)\ 731.3 \quad (b)\ 0.000442$.

Teaching Notes:

- Some of you may not see the difference between an exact value and a decimal approximation. You may think that 1/3=.333 is absolutely correct.
- The laws of exponents. You really need this throughout algebra.

Answers:

1. (a) < (b) > **2.** 

3. 7 **4.** (a) -1 (b) 20

5. $x=0,\ x=-2,\ x=2$ **6.** $(a)\ \frac{1}{100} \quad (b)\ \frac{1}{64} \quad (c)\ 4 \quad (d)\ \frac{x^{54}}{y^{24}} \quad (e)\ \frac{343x^{15}}{64y^{24}}$

7. $(a)\ 7.313\times 10^2 \quad (b)\ 4.42\times 10^{-4}$

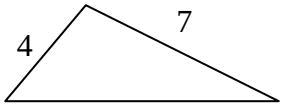
Mini-Lecture R.3
Geometry Essentials

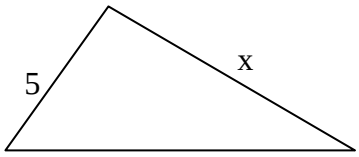
Learning Objectives:

1. Use the Pythagorean Theorem and its converse
2. Know geometry formulas
3. Understand congruent triangles and similar triangles

Examples:

1. The legs of a right triangle have lengths of 9 and 12. Find the hypotenuse.
2. Find the area of a triangle with height 7 inches and a base 8 inches.
3. Find the exact area and circumference of a circle with a radius of 4 meters.
4. Find the volume and surface area of a sphere of radius 9 centimeters.
5. Given that the following are similar triangles, find x.





Teaching Notes:

- Since the Pythagorean Theorem is used extensively throughout mathematics, make sure you understand it. Make sure you know it does not apply to triangles that are not right triangles.
- You need to know the formulas involving triangles and circles.

Answers:

1. 15

2. 28 square inches

3. $A = 16\pi \quad C = 8\pi$

4. $V = 972\pi \quad C = 324\pi$

5. $x = \frac{35}{4}$

Mini-Lecture R.4
Polynomials

Learning Objectives:

1. Recognize monomials and polynomials
2. Add and subtract polynomials
3. Multiply polynomials and know formulas for special products
4. Divide polynomials using long division

Examples:

1. State whether the expression is a polynomial. If it is, state the degree.

   $(a)\ 3x^3-7x+2 \quad (b)\ 3x-\dfrac{2}{x} \quad (c)\ \sqrt{2}x^2-3\pi \quad (d)\ \sqrt{2x}+5x$

2. Simplify each expression.

   $(a)\left(7x^3-6x^2+3x+8\right)-\left(6x^2-2x+7\right) \quad (b)\ \left(9x^5+2x^3+8x\right)+\left(5x^4-9x^3+9x^2\right)$

3. Simplify each expression.

   $(a)\ (x+2)\left(x^2+3x-2\right) \quad (b)\ (5x+3)(x-4) \quad (c)\ (x-2y)(x+y)$

   $(d)\ (x+8)^2 \quad (e)\ (x+2)(x-2) \quad (f)\ (x-2)^3$

4. Divide $5x^4-3x^2+8x+6$ by x^2+8.

Teaching Notes:

- It is important that you understand what a polynomial is.
- Being able to work with polynomials is essential for you to know. You must be able to simplify and multiply.
- Reinforce that $(x+y)^2 \neq x^2+y^2$. Make them use FOIL.

Answers:

1. (a) Yes; degree 3 (b) No (c) Yes; degree 2 (d) No

2. $(a)\ 7x^3-12x^2+5x+1 \quad (b)\ 9x^5+5x^4-7x^3+9x^2+8x$

3. $(a)\ x^3+5x^2+4x-4 \quad (b)\ 5x^2-17x-12 \quad (c)\ x^2-xy-2y^2$

   $(d)\ x^2+16x+64 \quad (e)\ x^2-4 \quad (f)\ x^3-6x^2+12x-8$

4. Quotient $=5x^2-43$; Remainder $=8x+350$

Mini-Lecture R.5
Factoring Polynomials

Learning Objectives:

1. Factor the difference of two squares, and sum and difference of two cubes
2. Factor perfect squares
3. Factor a second-degree polynomial : $x^2 + Bx + C$
4. Factor by grouping
5. Factor a second-degree polynomial: $Ax^2 + Bx + C,\ A \neq 1$

Examples:

1. Factor: $(a)\ x^2 - 36 \qquad (b)\ x^2 - 49 \qquad (c)\ x^3 - 64$

2. Factor: $(a)\ x^2 + 12x + 36 \qquad (b)\ x^2 - 16x + 64 \qquad (c)\ 81x^2 + 18x + 1$

3. Factor: $(a)\ x^2 - 10x + 24 \qquad (b)\ x^2 - 3x - 54 \qquad (c)\ x^2 + x - 30$

4. Factor: $(a)\ 18x^2 + 12x + 15x + 10 \qquad (b)\ 6x^2 + 21x + 8x + 28$

5. Factor: $(a)\ 6x^2 + 25x + 25 \qquad (b)\ 6x^2 - x - 12$

Teaching Notes:

- Factoring is a skill that is absolutely necessary for you to know, but one that so many cannot do adequately. Spend as much time as possible reinforcing this skill.
- Important to look for GCF and special cases first.

Answers:

1. $(a)\ (x-6)(x+6) \qquad (b)\ (x-7)(x+7) \qquad (c)\ (x-4)\left(x^2+4x+16\right)$

2. $(a)\ (x+6)^2 \qquad (b)\ (x-8)^2 \qquad (c)\ (9x+1)^2$

3. $(a)\ (x-6)(x-4) \qquad (b)\ (x-9)(x+6) \qquad (c)\ (x-5)(x+6)$

4. $(a)\ (3x+2)(6x+5) \qquad (b)\ (2x+7)(3x+4)$

5. $(a)\ (2x+5)(3x+5) \qquad (b)\ (2x-3)(3x+4)$

Mini-Lecture R.6
Synthetic Division

Learning Objectives:

1. Divide polynomials using synthetic division

Examples:

1. For the given expression, use synthetic division to find the quotient and the remainder.
   (a) $5x^4 - 6x^2 + 6x + 3$ divided by $x^2 + 9$
   (b) $9x^5 - 8x^2 + 2x + 8$ divided by $3x^3 - 1$
   (c) $3x^4 - 5x^3 + 5x + 3$ divided by $3x^2 + 3x + 2$

2. Use synthetic division to determine whether $x - c$ is a factor of the given polynomial.
   (a) $12x^3 - 15x^2 - 27x + 60;\ x - 2$
   (b) $2x^4 - x^3 - 4x + 2;\ x - \frac{1}{2}$
   (c) $9x^6 + 84x^3 + 96;\ x + 2$

Teaching Notes:

- This is not a hard thing for you to do, but you need to understand that it has significant importance in solving equations of degree greater than 2.
- Make sure you see how to use this process to factor.
- You might take this opportunity to mention the Remainder Theorem which will use this process.

Answers:

1. (a) $Q = 5x^2 - 51,\ \ R = 6x + 462$
   (b) $Q = 3x^2,\ \ R = -5x^2 + 2x + 8$
   (c) $Q = x^2 - \frac{8}{3}x + 2,\ \ R = \frac{13}{3}x - 1$

2. (a) No (b) Yes (c) Yes

Mini-Lecture R.7
Rational Expressions

Learning Objectives:

1. Reduce a rational expression to lowest terms
2. Multiply, divide, add, and subtract rational expressions
3. Simplify complex rational expressions

Examples:

1. Reduce each rational expression to lowest terms.

$(a)\ \dfrac{y^2-64}{6y^2-36y-96}$ $(b)\ \dfrac{x^2+3x-54}{6-x}$

2. Perform the indicated operation, and simplify the result.

$(a)\ \dfrac{3x+21}{9x^7}\cdot\dfrac{x}{x^2-49}$ $(b)\ \dfrac{x^2-3x-10}{x^2+3x-40}\cdot\dfrac{x^2+5x-24}{x^2+10x+16}$ $(c)\ \dfrac{\left(\dfrac{2x}{x^2-49}\right)}{\left(\dfrac{3x-15}{9x+63}\right)}$

$(d)\ \dfrac{\left(\dfrac{x^2+11x+18}{x^2-11x+18}\right)}{\left(\dfrac{x^2+7x-18}{x^2-7x-18}\right)}$ $(e)\ \dfrac{x^2}{5x-7}-\dfrac{4}{5x-7}$ $(f)\ \dfrac{5}{x-8}+\dfrac{x}{8-x}$

$(g)\ \dfrac{x}{x+9}+\dfrac{4x-5}{x-9}$ $(h)\ \dfrac{x}{x^2-8x+7}-\dfrac{x}{x^2-2x-35}$ $(i)\ \dfrac{4x}{x^2-9}-\dfrac{6}{x^2+x-12}$

3. Perform the indicated operation and simplify the result.

$(a)\ \dfrac{3+\dfrac{1}{x}}{6-\dfrac{1}{x}}$ $(b)\ \dfrac{\dfrac{x-6}{x+6}+\dfrac{x-1}{x+1}}{\dfrac{x}{x+1}-\dfrac{7x-3}{x}}$

Teaching Notes:

- Reducing rational expressions is always a problem.. You will try to cancel expressions that are not factored. See how a term must be factored in order to cancel it.
- There are two techniques shown for reducing a complex fraction. Method 1 is usually easier for the students.

Answers:

1. $(a)\ \dfrac{y+8}{6(y+2)}$ $\qquad (b)\ -(x+9)$

2. $(a)\ \dfrac{1}{3x^6(x-7)}$ $\qquad (b)\dfrac{x-3}{x+8}$ $\qquad (c)\ \dfrac{6x}{(x-5)(x-7)}$

$(d)\left(\dfrac{x+2}{X-2}\right)^2$ $\qquad (e)\ \dfrac{(x-2)(x+2)}{5x-7}$ $\qquad (f)\dfrac{5-x}{x-8}$

$(g)\ \dfrac{5x^2+22x-45}{(x-9)(x+9)}$ $\qquad (h)\ \dfrac{6x}{(x-7)(x-1)(x+5)}$ $\qquad (i)\ \dfrac{2(2x^2+5x-9)}{(x+3)(x-3)(x+4)}$

3. $(a)\ \dfrac{3x+1}{6x-1}$ $\qquad (b)\ \dfrac{-2x(x^2-6)}{(x+6)(6x^2+4x-3)}$

Mini-Lecture R.8
*n*th Roots ; Rational Exponents

Learning Objectives:

1. Work with *n*th roots
2. Simplify radicals
3. Rationalize denominators
4. Simplify expressions with rational exponents

Examples:

1. Simplify each expression. Assume all variables are positive.

   $(a)\sqrt[3]{216}$ $(b)\sqrt[3]{-343}$ $(c)\sqrt[4]{\dfrac{x^{10}y^{10}}{x^2y^6}}$

   $(d)5\sqrt{7}+6\sqrt{7}$ $(e)\left(\sqrt{7}+5\right)\left(\sqrt{7}-6\right)$ $(f)3\sqrt[3]{2}-8\sqrt[3]{128}$

2. Rationalize the denominator.

   $(a)\dfrac{4}{\sqrt{6}}$ $(b)\dfrac{\sqrt{3}}{2-\sqrt{2}}$ $(c)\dfrac{\sqrt{6}}{6-\sqrt{3}}$

3. Simplify each expression. Answers should have only positive exponents. Assume all variables are positive.

   $(a)64^{2/3}$ $(b)(-216)^{1/3}$ $(c)\left(\dfrac{64}{512}\right)^{2/3}$

   $(d)\left(x^4y\right)^{1/3}\left(xy^4\right)^{2/3}$ $(e)\left(8x^3y^{-1/2}\right)^{2/15}$ $(f)\left(4x^2y^{-1/3}\right)^{3/14}$

Teaching Notes:

- Make sure you understand how radicals can be combined. $2+3\sqrt{5}\neq 5\sqrt{5}$
- It is essential you understand how to simplify exponential expressions. See the properties of exponents.

Answers:

1. $(a)6$ $(b)-7$ $(c)\,x^2y$ $(d)11\sqrt{7}$ $(e)-23-\sqrt{7}$ $(f)-29\sqrt[3]{2}$

2. $(a)\dfrac{2\sqrt{3}}{3}$ $(b)\dfrac{2\sqrt{3}+\sqrt{6}}{2}$ $(c)\dfrac{2\sqrt{6}+\sqrt{2}}{11}$

3. $(a)16$ $(b)-6$ $(c)\dfrac{1}{4}$ $(d)\,x^2y^3$ $(e)\dfrac{4^{1/5}x^{2/5}}{y^{1/15}}$ $(f)\dfrac{8^{1/7}x^{3/7}}{y^{1/14}}$