Math 12 **P.2: Integer Exponents**

Integer Exponents: $a^n = a \bullet a \bullet a \bullet a \bullet\bullet\bullet a$ n times.

<table>
<tr><td colspan="3">Rules of Exponents
For any nonzero numbers a and b and any positive integer n,</td></tr>
<tr><td rowspan="3">Definitions:
$a^1 = a$
and
$a^0 = 1$
and
$a^{-n} = \frac{1}{a}$</td><td>Product: $a^m \bullet a^n = a^{m+n}$</td><td>Power of a Product:
$(a \bullet b)^n = a^n \bullet b^n$</td></tr>
<tr><td>Quotient: $\frac{a^m}{a^n} = a^{m-n}$</td><td rowspan="2">Power of a Quotient:
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
and
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$</td></tr>
<tr><td>Power: $\left(a^m\right)^n = a^{m \bullet n}$</td></tr>
</table>

An exponential expression is considered **simplified** when:

1. each base appears only once,
2. no negative or zero exponents are used, and
3. no power is raised to a power.

Examples: Simplify the expression and eliminate any negative exponent(s).

1. $-8\left(xy^3\right)^4$

2. $\left(-2x^{-3}y\right)^2$

3. $\left(\frac{-2xy^2}{y}\right)^3$

4. $\frac{15x^6y^{-2}}{3x^4y^{-5}}$

5. $\left(5x^3\right)^2\left(\frac{1}{125}x^4\right)^2$

6. $\left(2u^2v^3\right)^3\left(3u^3v\right)^{-2}$

Application: Scientific Notation: $c \times 10^n$, where $1 \le c < 10$.

Examples: Express the number in each statement in scientific notation

1. The mass of the earth is about 5,970,000, 000, 000, 000, 000, 000, 000 kg.

2. The diameter of and electron is about 0.0000000000004 cm.

3. $\left(2.4 \times 10^{-4}\right)\left(1.6 \times 10^{13}\right)$

4. $\dfrac{\left(2.4 \times 10^{-2}\right)}{\left(1.2 \times 10^{-7}\right)}$

P.4: Factoring Polynomials

To Factor a Polynomial completely:

1. Always look for and factor out the GCF before doing anything else.
2. Determine the number of terms in the polynomial and try factoring as follows:
   a. If there are 2 terms (a binomial):
      i. Is it a Difference of 2 squares?
      ii. Is it a Sum or Difference of 2 cubes?
   b. If there are 3 terms (a trinomial):
      i. Can you factor into ($\pm$)($\pm$)? Watch the signs!
   c. If there are 4 terms:
      i. Factor by grouping method
3. Make sure that all factors have no factors in common – no GCG, no difference of squares, and no sum/difference of cubes.
4. Check by multiplying.

Examples:

1. $6x^3 + 24x$

2. $7y^3 - 21y^2 + 14y$

3. $y^2 + 36$

4. $5x^3 - 20x^2 + 20x$

5. $3r^3 - 27r$

6. $2y^5 - 128y^2$

7. $y^9 - y^5$

8. $3x^4y^2 - 3x^2y^2$

9. $x^3 - 3x^2 + 4x - 12$

Math 12 **Chapter P Supplement Problems**

P.2 Integer Exponents

Evaluate. (Do not use a calculator.)

1. $\dfrac{1}{6^{-2}}$
2. $\dfrac{3^{-2}}{5}$
3. $6^{2}\cdot 3^{-1}$
4. $-17^{0}+6^{-2}$
5. $\left(3^{-2}-3^{-1}\right)^{-2}$
6. $\dfrac{5}{2^{-1}}+\dfrac{5^{-1}}{2}+\left(\dfrac{5}{2}\right)^{-1}$
7. $\dfrac{8^{-1}}{3-4^{-2}}$
8. $\dfrac{7^{0}-5^{-1}}{6^{-1}+3^{-2}}$

Simplify.

9. $\left(3a^{6}b^{-5}\right)^{-4}$
10. $\left(\dfrac{mn^{-4}}{m^{5}n^{-2}p}\right)^{-7}$
11. $\left(6c^{-10}d\right)^{-2}\left(3cd^{-7}\right)^{3}$
12. $\dfrac{5\left(2a^{-7}c\right)^{3}}{\left(10a^{8}b^{6}c^{-10}\right)^{-2}}$
13. $\left(\dfrac{xy^{6}z^{-2}}{x^{-2}z}\right)^{-5}\left(\dfrac{y^{-2}}{xy^{-10}z^{-3}}\right)^{6}$
14. $\dfrac{\left(4ab^{-5}c^{-2}\right)^{2}}{\left(2a^{-2}b^{9}\right)^{-1}}\cdot\left(\dfrac{10a^{7}c^{-5}}{a^{-7}b^{4}c^{-2}}\right)^{-3}$

P.4 Factoring and Simplifying Expressions

Factor the expression completely.

1. $x^{4}-5x^{2}-36$
2. $x^{6}+3x^{5}-10x^{4}$
3. $x^{6}-4x^{3}-5$
4. $\left(x-3\right)^{2}-25y^{10}z^{12}$
5. $1000a^{3}+27b^{6}$
6. $x^{\frac{5}{2}}-9x^{\frac{1}{2}}$
7. $8x^{8/3}-x^{-1/3}$
8. $2x^{4/3}-7x^{1/3}-4x^{-2/3}$
9. $2x^{\frac{2}{3}}-3x^{\frac{1}{3}}-20$
10. $x^{-3/2}+2x^{-1/2}+x^{1/2}$
11. $6x\left(x+4\right)-5\left(x+4\right)^{2}$
12. $\left(2x-7\right)^{5}\left(x-8\right)^{30}+\left(2x-7\right)^{4}\left(x-8\right)^{31}$
13. $10x^{15}\left(x-6\right)^{-4}-8x^{14}\left(x-6\right)^{-3}$
14. $3x^{-1/2}\left(4x+3\right)^{5/4}+x^{3/2}\left(4x+3\right)^{1/4}$

Simplify.

15. $\dfrac{5x\left(x+7\right)^{4}-2x^{2}\left(x+7\right)^{3}}{\left(x+7\right)^{11}}$
16. $\dfrac{5\left(2x-1\right)^{-1/3}+6\left(2x-1\right)^{2/3}}{2x-1}$

Answers for P.2

1. 36 2. $\frac{1}{45}$ 3. 12 4. $-\frac{35}{36}$ 5. $\frac{81}{4}$ 6. $\frac{21}{2}$

7. $\frac{2}{47}$ 8. $\frac{72}{25}$ 9. $\frac{b^{20}}{81a^{24}}$ 10. $m^{28}n^{14}p^{7}$ 11. $\frac{3c^{23}}{4d^{23}}$ 12. $\frac{4000b^{12}}{a^{5}c^{17}}$

13. $\frac{y^{18}z^{33}}{x^{21}}$ 14. $\frac{4b^{11}c^{5}}{125a^{42}}$

Answers for P.4

1. $(x^2+4)(x+3)(x-3)$ 2. $x^4(x+5)(x-2)$ 3. $(x^3-5)(x+1)(x^2-x+1)$

4. $(x-3+5y^5z^6)(x-3-5y^5z^6)$ 5. $(10a+3b^2)(100a^2-30ab^2+9b^4)$

6. $x^{\frac{1}{2}}(x+3)(x-3)$ 7. $x^{-1/3}(2x-1)(4x^2+2x+1)$ 8. $x^{-2/3}(2x+1)(x-4)$

9. $(2x^{1/3}+5)(x^{1/3}-4)$ 10. $x^{-3/2}(1+x)^2$ 11. $(x+4)(x-20)$

12. $3(2x-7)^4(x-8)^{30}(x-5)$ 13. $2x^{14}(x-6)^{-4}(x+24)$

14. $x^{-1/2}(4x+3)^{1/4}(x^2+12x+9)$ 15. $\frac{x(3x+35)}{(x+7)^8}$ 16. $\frac{12x-1}{(2x-1)^{\frac{4}{3}}}$

Math 12 **P.5: Rational Expressions**

A **rational expression** is a quotient of two polynomials, $\frac{N(x)}{D(x)}$. Remember $D(x) \neq 0$

To **simplify** a rational expression you need to factor both the numerator and denominator and then remove any common factors (cancel).

To **determine the domain** of a ration expression, set $D(x) = 0$ and solve. These are the values that are removed from the domain of any polynomial, $(-\infty, \infty)$.

Examples: a) Reduce each rational expression to lowest terms. b) Identify all numbers that must be excluded from the domain of the given rational expression.

1. $\dfrac{x^2 - x - 2}{x^2 - 1}$

2. $\dfrac{2x^3 - x^2 - 6x}{2x^2 - 7x + 6}$

Multiplication and Division of Rational Expressions

To **multiply:** First, factor both the numerator and denominator, cancel common factors, and then leave your answer in factored form.

To **divide:** Change problem to multiplying by the reciprocal of the divisor, and then follow the steps above.

Examples: Multiply or divide as indicated. Simplify, and leave the numerator and denominator in your answer in factored form.

1. $\dfrac{x^2 - x - 12}{x^2 - 9} \bullet \dfrac{3 + x}{4 - x}$

2. $\dfrac{4y^2 - 9}{2y^2 + 9y - 18} \div \dfrac{2y^2 + y - 3}{y^2 + 5y - 6}$

Addition and Subtraction of Rational Expressions

You **must** have common denominators to add or subtract rational expressions. (See box pg. 55)

To add/subtract:

1. Factor denominators of each term to determine the LCD.
2. Rewrite each term using the new LCD.
3. Add or subtract the numerators. When subtracting, don't forget to distribute the negative through the numerator following the subtraction sign. Also, don't lose the denominator.

Examples: Add or subtract as indicated. Simplify, and leave the numerator and denominator in your answer in factored form.

1. $\dfrac{2x}{x^2-4}+\dfrac{x}{x+2}$

2. $\dfrac{2}{x+3}-\dfrac{1}{x^2+7x+12}$

3. $\dfrac{x}{x^2+x-2}-\dfrac{2}{x^2-5x+4}$

4. $\dfrac{1}{x+1}-\dfrac{2}{\left(x+1\right)^2}+\dfrac{3}{x^2-1}$

Complex Fractions
A rational expression that contains a rational expression in the numerator or denominator (or both)

There are two methods of simplifying complex fractions. (See box on pg. 57)

Examples: Simplify the compound fraction, and leave the numerator and denominator in your answer in factored form.

1. $$\frac{2-\frac{2}{x+1}}{1+\frac{2}{x+1}}$$

2. $$\frac{\frac{1}{x-a}+\frac{1}{x+a}}{\frac{x}{x-a}-\frac{a}{x+a}}$$

3. $$\frac{x^{-1}+y^{-1}}{\left(x+y\right)^{-1}}$$

Math 12 **P.6: Rational Exponents and Radicals**

Definition: $\sqrt{a} = b$ means (1) $b^2 = a$ and (2) $b \geq 0$. $\sqrt{}$ is the principal square root.

The domain for the expression $\sqrt{x}$ is $x \geq 0$ or $[0, \infty)$.

<table>
<tr><th colspan="3">Properties of Roots and Rational Exponents</th></tr>
<tr><td rowspan="2">Definitions:
For any real number x,
$\sqrt{x^2} = |x|$
and
$\left(\sqrt{x}\right)^2 = x$</td><td>Product of Square Root:
$\sqrt{a \bullet b} = \sqrt{a} \bullet \sqrt{b} \quad a \geq 0, b \geq 0$</td><td>Product of nth Root:
$\sqrt[n]{a \bullet b} = \sqrt[n]{a} \bullet \sqrt[n]{b}$</td></tr>
<tr><td>Quotient of Square Root:
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad a \geq 0, b > 0$</td><td>Quotient of Square Root:
$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$</td></tr>
<tr><td>Simplifying $\sqrt[n]{a^n}$
If n is odd, then
$\sqrt[n]{a^n} = a$
If n is even, then
$\sqrt[n]{a^n} = |a|$</td><td colspan="2">The Principal nth Root of a Real Number

1. If $a > 0$, then $\sqrt[n]{a} = b$, provided that $b^n = a$ and $b > 0$.
2. If $a < 0$ and n is odd, then $\sqrt[n]{a} = b$, provided that $b^n = a$.
3. If $a < 0$ and n is even, then $\sqrt[n]{a}$ is not a real number.</td></tr>
<tr><td>Rational Exponents:
$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$</td><td colspan="2">Definition: For any real number a and any integer $n > 1$,
$a^{\frac{1}{n}} = \sqrt[n]{a}$.
When n is even and $a < 0$, $\sqrt[n]{a}$ and $a^{\frac{1}{n}}$ are not real numbers</td></tr>
</table>

To **simplify** a radical expression remove perfect squares that are factors of the radicand from under the radical sign.

It is customary to rewrite a quotient involving square roots so that no radical appears in the denominator. The process to eliminate square roots in a denominator is called **rationalizing the denominator.**

1. $\sqrt{27x^5}$ **2.** $\sqrt{\frac{3}{50}}$ **3.** $\sqrt[5]{x^{15}y^5}$ **4.** $\frac{4x^3}{\sqrt[7]{16x^4}}$

When adding or subtracting you must have **like radicands**.

4. $\sqrt{75x^7} + x^3\sqrt{48x}$

5. $x^5\sqrt[3]{24x^2} - \sqrt[3]{81x^{17}}$

Simplify each expression and eliminate any negative exponent(s).

6. $x^{\frac{1}{2}} \bullet x^{\frac{2}{7}}$

7. $\left(\dfrac{375}{8}\right)^{-\frac{1}{3}}$

8. Convert each radical expression to it rational exponent form and then find the product, stating your answer as a single radical. $\sqrt[4]{2x^2y} \bullet \sqrt[12]{5x^5y^2}$

Math 12 **Chapter P Supplement Problems**

P.5 Rational Expressions

Simplify the compound fraction.

1. $\dfrac{x^{-3}+y^{-3}}{x^{-2}-y^{-2}}$

2. $\dfrac{5(x-1)^{-1}-2(x+1)^{-1}}{x(x-1)^{-1}+(x+1)^{-1}}$

P.6 Rational Exponents and Radicals

Rationalize the denominator of each of the following.

1. $\dfrac{5x^4}{\sqrt[7]{x^3}}$

2. $\dfrac{16}{\sqrt[4]{8x}}$

3. $\dfrac{y}{\sqrt{7}-\sqrt{y}}$

4. $\dfrac{9(x-y)^7}{\sqrt{x}+\sqrt{y}}$

Rationalize the numerator of each of the following.

5. $\dfrac{\sqrt{5}-\sqrt{3}}{-4}$

6. $\dfrac{\sqrt{3}-\sqrt{7x}}{\sqrt{y^3}}$

7. $\dfrac{\sqrt{x+h}+\sqrt{x}}{h}$

8. $\dfrac{\sqrt{x}-\sqrt{x-h}}{h\sqrt{x}\sqrt{x-h}}$

9. $\sqrt{x^2+5}-x$

Answers:

P.5: **1.** $\dfrac{y^2-xy+x^2}{xy(y-x)}$ **2.** $\dfrac{3x+7}{x^2+2x-1}$

P.6: **1.** $5x^3\sqrt[7]{x^4}$ **2.** $\dfrac{8\sqrt[4]{2x^3}}{x}$ **3.** $\dfrac{y(\sqrt{7}+\sqrt{y})}{7-y}$ **4.** $9(x-y)^6(\sqrt{x}-\sqrt{y})$ **5.** $\dfrac{-1}{2(\sqrt{5}+\sqrt{3})}$

6. $\dfrac{3-7x}{y\sqrt{y}(\sqrt{3}+\sqrt{7x})}$ **7.** $\dfrac{1}{\sqrt{x+h}-\sqrt{x}}$ **8.** $\dfrac{1}{x\sqrt{x-h}+(x-h)\sqrt{x}}$ **9.** $\dfrac{5}{\sqrt{x^2+5}+x}$